MANAGING CAPITAL FLOWS IN THE PRESENCE OF EXTERNAL RISKS

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Abstract. The cross-country capital flows that originated from the recent global financial crisis have sparked a renewed interest from policy makers in the use of macroprudential instruments to prevent and reduce the effects of sudden capital reversals. The recent theoretical studies of macroprudential policy in borrowing-constrained economies have not taken into account the implications of external uncertainty for the design of optimal policy, something that deems very relevant given the high volatility in international financial markets. In this paper, we study optimal policy responses to shocks in the mean and volatility of the external interest rate in a small open economy with an occasionally binding borrowing constraint. In the model, sudden stops in external financing arise endogenously and are accompanied with sharp declines in asset prices and consumption. We show that the modeled evolution of interest rates around episodes of sudden stops is consistent with the empirical evidence for a group emerging markets. We solve the problem of a benevolent social planner and show numerically that: (i) his policy is indeed contingent on the level and volatility of the external interest rate shocks, and (ii) the intensity of the planner's policy is non-monotonic with respect to the volatility of external shocks. We argue that the planner takes into account two factors to determine the size of his intervention: whether a sudden stop is likely to occur in the near future, and how large are the pecuniary externalities derived from the households' borrowing decisions.

1. Introduction

The consequences of the global financial crisis of 2008 have led to a reemergence in the debate regarding the optimal use of different policy instruments to reduce the risks carried by large and volatile capital flows across countries. Recent policy proposals have considered the use of instruments such as capital controls or other restrictions on capital flows to reduce their risks, and new theoretical contributions have established the grounds on which the use of these different instruments rest. However, even though the risks associated with higher

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volatility of capital flows—in part due to higher uncertainty in the world economy—have been identified as relevant, the recent theoretical literature has not yet studied the influence of external shocks in the design of optimal policy.

In this paper, we show how optimal policy should respond to external shocks by introducing these into a benchmark model of sudden stops that has been recently used to study the design of optimal macroprudential policy. The baseline theoretical framework considers a small open economy that faces an external borrowing limit that depends on the value of a domestic non-tradable asset. We introduce external risk by means of shocks to the mean and variance of interest rates at which the small country borrows and lends, and we ask how a benevolent social planer would set an optimal policy in response to external risks. We show that the optimal policy is contingent not only on the level, but also on the volatility of external shocks, and that the macroprudential tax on debt introduced by the social planner is non-monotonic with respect to the level of the volatility of the external shocks. Our results shed light on the optimal use of macroprudential controls in a particularly relevant moment, since many emerging economies have shown recent concerns about the volatility in global markets, partly due to the uncertainty in the decisions of advanced economies regarding their countercyclical policies.

The use of policy tools to manage capital flows across countries has repeatedly been at the center of the debate in international macroeconomics. Until very recently, the benefits of liberalizing capital flows were considered greater than their intrinsic risks; thus, under this view, no management of capital flows should be called upon. However, the global financial crisis of 2008 and the associated transfers of capital across countries have generated a reemergence of the interest in the policy and academic agenda on the use of different policy tools to either prevent or minimize the costs associated with capital flows. A number of policy studies that call for a more active management of capital flows have been published since the crisis.[1] These studies identify two sources of risk associated with capital flows across countries: (i) the actual size, and (ii) the volatility of these flows. In particular, the policy agenda has identified both sources of risk as posing significant policy challenges for all countries, but especially for emerging economies. Hence, a new policy paradigm has emerged that includes policies such as capital controls or other type of restrictions on capital flows as potential policy tools when it comes to prevent or minimize the ex post costs associated with the risks carried by capital flows.

Paralleling the changes in the policy agenda, a theoretical literature has emerged that provides theoretical grounds, in terms of a country's welfare, for the implementation of many of the aforementioned policy recommendations.[2] A theoretical framework based on dynamic-stochastic general equilibrium models traditionally exploited to study the positive side of large and abrupt capital outflows in emerging economies—also known as the sudden stops

[1]See Ostry et al. (2011) and Dell'Ariccia et al. (2012) as analytical background for IMF (2012), the organization's institutional view, and the policy proposals in IMF (2013), Chapter 4.
[2]Korinek (2011) and Korinek and Mendoza (2013) provide extensive surveys of this literature.

phenomenon—has emerged as a benchmark framework to analyze the normative aspects of optimal policy in the face of the risks carried by capital flows.[3] In particular, the rationale for policy intervention in this framework emerges due to the presence of a pecuniary externality in the domestic agents' borrowing decisions. The externality arises from the fact that the external borrowing constraint of the economy hinges on an endogenously determined price—either the price of an asset or the real exchange rate—that depends itself upon the aggregate level of external indebtedness.

Most theoretical frameworks used to study capital flows and optimal policy share two fundamental features. First, the shocks leading to a sudden stop—which is typically interpreted as a binding borrowing constraint—are either particular to a country's fundamentals or zero-probability events exogenously imposed to the model. Hence, optimal policy in these studies does not depend on potentially relevant external shocks. Second, the possibility of a sudden stop relies most importantly on large capital inflows that increase the level of a country's leverage, and this has come to be appreciated in the literature as the economy "overborrowing" relative to a social planner's borrowing decisions. Thus, the normative implications of these variants of the model are most directly related to the policy challenge imposed by the size of capital flows rather than their volatility, the latter being recently emphasized in the policy arena.

The literature on emerging economies has documented that these countries not only face the risk of a sudden stop due to weak fundamentals, but also significant risks associated to external shocks, which are independent of a country's fundamentals.[4] A strand of the literature has focused on the effects of international interest rates on these countries, and has by now clearly documented that there are significant effects of this type of shocks on real economic activity in emerging markets. By considering the shocks to interest rates at which these economies borrow as driven by external factors, these studies have identified that not only the first, but also the second moment of these shocks matter for emerging market business cycles.[5] Moreover, recent studies have also shown empirically that these shocks have direct effects on capital flows across countries.[6]

In addition, there exists an empirical association between sudden capital flow reversals and external interest rate volatility. Reyes-Heroles and Tenorio (2015) document the empirical patterns of interest rates and output faced by emerging markets around the beginning of sudden stops. Their main findings are that: (i) sudden stops are preceded by periods of below-normal interest rates, which rise when the sudden stop occurs, and revert to their normal levels in the following years; (ii) sudden stops are preceded by periods of slowly-increasing interest rate volatility, which spikes sharply both at the beginning and the end of the sudden stop; and (iii) sudden stops are preceded by economic expansions, which abruptly

[3] Mendoza and Smith (2002) and Mendoza (2010) explore the positive aspects of this framework.
[4] See Mackowiak (2007) and Chang and Fernández (2013).
[5] See Uribe and Yue (2006), Neumeyer and Perri (2005) and Fernández-Villaverde et al. (2011).
[6] See Ahmed and Zlate (2013).

turn into output drops at the beginning of the episode, and are followed by slow recoveries. Given the associations between output, interest rates, and volatility with capital flow reversals in the data, it becomes relevant to consider the role of these external factors in the design of optimal policy.[7]

For this reason, in this paper we introduce these types of shocks to the simplest benchmark theoretical framework that has been used to study the qualitative and quantitative features of optimal policy regarding capital flows, and we analyze the quantitative implications of these shocks. We do so by considering a variant of the small open economy model proposed by Jeanne and Korinek (2010) and Bianchi and Mendoza (2013). We extend their model by letting the interest rate at which the economy borrows follow a stochastic process with time-varying volatility. In our model, a small open economy is populated by a continuum of households, whose only source of income is the payoff of a risky asset. The asset's shares cannot be traded across borders, but the households can lend or borrow from abroad in the form of non-contingent riskless bonds. Borrowing is subject to a collateral constraint, and the amount of collateral available depends on the value of the households' holdings of the risky asset. The households also face a refinancing risk, because the international interest rate is stochastic, so they take this into account when making consumption and saving decisions. A sudden stop occurs when a long enough series of negative shocks drives the households to borrow up to the point where the borrowing constraint binds. This forces an abrupt deleveraging of the households, which reduces current consumption and causes a drop in asset prices, further reducing the value of collateral and tightening the borrowing constraint. By simulating the model, we show that the evolution of the interest rate during sudden stop events is similar to the empirical event windows that we document for a sample of emerging markets.

After setting up the model and discussing the main features of the competitive equilibrium, we compare the equilibrium allocation to that of a social planner that internalizes the effects of borrowing on the price of the asset. Even though the planner internalizes the effects of borrowing on the asset price and on the borrowing constraint, he cannot choose a price directly and acts according to this price being consistent with equilibrium conditions. We solve the model numerically using global methods, and investigate the implications of the external shocks on optimal policy. The use of global methods is necessary in this type of models in order to fully characterize the nonlinearities that arise in the region where the collateral constraint binds.

Even though we consider a simple model that lacks some of the mechanisms that the literature has identified as relevant in order for external shocks to significantly affect business cycles, the simplicity of the model clarifies the central mechanisms at play that help us answer

[7]Furthermore, recent studies have shown that an environment of high volatility is most likely to continue for emerging economies throughout the unwinding of the countercyclical policies implemented in advanced economies, which stresses the importance of considering volatility shocks in policy (e.g., Aizenman et al., 2014; Eichengreen and Gupta, 2014).

one key question in the paper: does greater external volatility call for higher taxes on capital flows? Since this model allows us to focus on the effects of the pecuniary externality, we disregard other mechanisms that imply a negative effect of higher volatility on the economy. Hence, our results can be interpreted as a lower bound on the responsiveness of the social planner's optimal policy with respect to external shocks.

The main results of our numerical exercises can be summarized as follows: (i) the optimal policy is indeed contingent on the size of external shocks; (ii) even in this very simplified framework, optimal policy depends on the volatility of the external shock; and (iii) the level of capital flow taxation that decentralizes the planner's allocation is non-monotone in the level of external volatility. This last result should be underscored, as common intuition tells us that higher volatility should lead to more stringent capital controls, as the probability of a binding collateral constraint increases. However, as we discuss in the paper, this intuition does not take into account the effects of interest rates on the pecuniary externality, which might have an offsetting effect on the planner's decision.

1.1. **Related literature.** This paper is mainly related to a relatively recent strand of literature that explores optimal policy, in particular the use of capital controls, to mitigate the risks associated with capital flows across countries. The methodology we follow is most closely related to Jeanne and Korinek (2010). They focus on a simple framework in order to analyze the implications of the pecuniary externality that drives the amplification mechanism that opens up the possibility for second-best type of policies. This amplification mechanism was initially introduced to the positive study of sudden stops in Mendoza (2002), Mendoza and Smith (2002) Mendoza and Smith (2006) and Mendoza (2010). However, the events associated with the global financial crisis of 2008 have fostered the studies focusing on the normative aspects of these mechanisms. Within this framework, the literature has focused on two different aspects of optimal policy, either its "prudential" features, in the sense that policy is undertaken ex ante in order to reduce the probability of a crisis, or its ex post characteristics, once a crisis has occurred. Jeanne and Korinek (2010), Bianchi (2011), Bianchi and Mendoza (2011) and Bianchi and Mendoza (2013) focus on the former, while Benigno et al. (2011) and Benigno et al. (2013b) focus on the latter. Most recently, other studies like Jeanne and Korinek (2013) and Benigno et al. (2013a) have focused on the use of both, ex ante as well as ex post policies in order to mitigate the risks associated to capital flows.

Another strand of the literature has studied the effects of external shocks on emerging market business cycles. More precisely, most of these studies have looked at shocks to the interest rate at which emerging markets borrow as a potential source of variation in real economic activity. Uribe and Yue (2006) and Neumeyer and Perri (2005) study the effect of interest rate shocks on emerging markets business cycles. Fernández-Villaverde et al. (2011) show that not only the first, but also the second moment of the shocks to interest rates have implications on real economic activity in emerging markets. This paper follows the methodology of these studies to introduce external shocks in our model of sudden stops. In a recent study, Carrière-Swallow and Céspedes (2013), have further emphasized that global uncertainty has important effects on real economic activity in emerging economies.

The rest of this paper is organized as follows. In Section 2 we introduce the theoretical model of a small open economy that is borrowing constrained, and that faces domestic and external risks. We describe the competitive equilibrium and discuss the presence of a pecuniary externality that motivates the intervention of a social planner to increase welfare in the economy. In Section 3 we present the results of our numerical exercises. We show that the dynamics of interest rates around episodes of sudden stop in the model are consistent with their empirical counterparts. Moreover, we explain how the optimal response of the planner is shaped by the possibility of future binding borrowing constraints and by the size of pecuniary externalities. In Section 4 we conclude with the main implications of our exercise for macroeconomic policy.

2. A model of endogenous sudden stops with external interest rate risk

2.1. **Framework.** Our framework is closely related to Jeanne and Korinek (2010) and Bianchi and Mendoza (2013). There is an open economy inhabited by a continuum of unit measure of identical households that have preferences for streams of a consumption good, c_t, given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where u is an increasing, concave, and differentiable function that satisfies the usual Inada conditions.

There is a Lucas tree that yields a stochastic flow of consumption goods of $d_t = d \exp(z_t)$ per period. The flow of goods provided by the tree can be traded period by period with the rest of the world, but the stocks of the tree can only be held by domestic owners. A possible explanation is that this arrangement arises from drastic (unmodeled) asymmetries of information between domestic managers and international investors that impede foreigners to earn profit from holding stocks of the tree. We denote by q_t the market value of the tree at time t, and by s_t the holdings of the asset chosen by the representative household.

Households have access to debt financing in international financial markets in order to smooth their consumption and fund their stock purchases. The bonds issued by households in international markets have a maturity of one period, and they pay an exogenous gross return of $R_t = R \exp(r_t)$. We let the external interest rate have a stochastic transition, but debt contracts are locally risk free: a household knows at time t the interest rate that it must pay next period for its outstanding bonds, but it does not know the interest rate that it will face next period if it decides to refinance its stock of debt.

Motivated by the findings of Reyes-Heroles and Tenorio (2015), we allow for contemporaneous correlation and dynamic feedbacks between the exogenous output and interest rate processes. The random vector $(z_t, r_t)'$ has the following VAR specification:

$$\begin{pmatrix} z_t \\ r_t \end{pmatrix} = A_0 + A_1 \begin{pmatrix} z_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^r \end{pmatrix}. \quad (1)$$

The draws of the shock vector $(\epsilon_t^z, \epsilon_t^r)'$ are independent across time, and they have a Gaussian distribution with zero mean and a covariance matrix that has itself a stochastic evolution:

$$\Sigma_t = \begin{pmatrix} (\sigma^z)^2 & \rho \cdot \sigma^z \cdot \sigma_t^r \\ \rho \cdot \sigma^z \cdot \sigma_t^r & (\sigma_t^r)^2 \end{pmatrix}.$$

As in Reyes-Heroles and Tenorio (2015), we allow the external interest rate volatility to take on two values, $\sigma_t^r \in \{\sigma_L^r, \sigma_H^r\}$, with $\sigma_H^r > \sigma_L^r > 0$. The switching between these regimes is governed by a first-order Markov process with transition matrix Π.

Let us denote by b_t the face value of bonds that are held by the households at the beginning of period t. Throughout the paper, we follow the convention that a positive b_t represents savings of the households overseas, whereas negative positions represent external household debt. The time t budget constraint faced by a household is:

$$c_t + q_t s_{t+1} + \frac{b_{t+1}}{R_t} = (q_t + d_t) s_t + b_t. \quad (2)$$

The key friction in this economy is that the amount of borrowing that households can undertake is limited by the value of their asset holdings. More specifically, the market value of debt issued by a representative household at time t, $-\frac{b_{t+1}}{R_t}$, is constrained to be less than or equal to the valuation of their holdings of stocks of the tree, $q_t^c s_{t+1}$, multiplied by a constant κ that determines how stringent the financial frictions are:

$$-\frac{b_{t+1}}{R_t} \leq \kappa q_t^c s_{t+1}. \quad (3)$$

We are making explicit the fact that the price used to value asset holdings as collateral at time t, q_t^c, is not necessarily the same as the market price, q_t. In Appendix A.1, we provide a microeconomic foundation of the collateral constraint that is based in contractual imperfections, as is common in the literature of financial frictions (e.g., Kiyotaki and Moore, 1997; Bernanke et al., 1999). The main idea is that within each period, there is a time in which households can divert a fraction $(1-\kappa)$ of the assets previously posted as collateral, sell them off at the prevailing price q_t^c, and default on their outstanding loans. After this, the foreign lender is entitled to the remaining fraction κ of collateral assets, which must be sold in the domestic market at the prevailing price q_t^c. In the appendix, we show that the market price of the tree and its resale value need not be the same, and we also derive the relationship that has to hold in equilibrium between them.

2.2. **Competitive equilibrium.** A competitive equilibrium is a sequence of allocations $\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}$ for every household and a prices of the tree $\{q_t, q_t^c\}_{t=0}^{\infty}$ (market and collateral valuations) such that households optimize their utility, subject to the budget and borrowing constraints, and the market for stocks of the tree clears. Given the fact that all the households are identical and they only face aggregate shocks, market clearing implies that $s_t = 1$ in every period.

We rewrite the problem of the representative household in recursive form in order to highlight the role of pecuniary externalities in the competitive equilibrium. The aggregate states in the household's problem are the aggregate level of savings B, and the current realization of the stochastic shocks, which we denote $X \equiv (z, r, \sigma^r)$. The individual states of a household are its holdings of bonds b, and stocks of the tree s. We denote by $V(b, s, B, X)$ the value of the problem for a household with portfolio (b, s) when the aggregate states are B and X. Households take as given a perceived law of motion for aggregate bonds, $B' = \mathcal{B}(B, X)$, in order to form expectations on future prices. Then, the Bellman equation of the problem is:

$$V(b, s, B, X) = \max_{c, b', s'} u(c) + \beta \mathbb{E}[V(b', s', B', X')|X] \tag{4}$$

subject to:

$$\begin{aligned} c + \mathcal{Q}(B, X)s' + \frac{b'}{R(X)} &= [\mathcal{Q}(B, X) + d(X)]s + b, \\ -\frac{b'}{R(X)} &\leq \kappa \mathcal{Q}^c(B, X)s', \\ B' &= \mathcal{B}(B, X). \end{aligned}$$

In the previous expression, $\mathcal{Q}(B, X)$ is the market value of the tree, and $Q^c(B, X)$ is the value of the asset when employed as collateral. These two prices are determined in equilibrium and depend on the aggregate states of the economy. In a recursive competitive equilibrium, it must be the case that $\mathcal{B}$ is consistent with optimal individual decision rules, and that $\mathcal{Q}$ and $\mathcal{Q}^c$ ensure the clearing of the market for stocks of the tree, in the different trading cycles.

In Appendix A.2 we show that the solution to the household's problem satisfies the following Euler equations for bonds and stocks of the tree, respectively:

$$\begin{aligned} & u'(c(b, s, B, X)) - \mu(b, s, B, X) \\ & \qquad = R(X)\beta \mathbb{E}\left\{u'(c(b', s', \mathcal{B}(B, X), X'))|X\right\}, \\ & \mathcal{Q}(B, X)u'(c(b, s, B, X)) \cdot \left(1 + \frac{\kappa \mu(b, s, B, X)}{u'(c(b, s, B, X))}\right)^{-1} \\ & \qquad = \beta \mathbb{E}\left\{u'(c(b', s', \mathcal{B}(B, X), X'))[\mathcal{Q}(\mathcal{B}(B, X), X') + d(X')]|X\right\}, \end{aligned}$$

where $\mu \geq 0$ is the multiplier on the borrowing constraint. The left hand side of the Euler equation for bonds is the marginal cost of saving an additional unit of consumption good at time t: the household loses utility $u'(c_t)$ in the margin and, if the borrowing constraint is binding, an additional unit of saving relaxes the constraint, with a shadow value of μ_t, thus reducing the marginal cost of saving. The right hand side represents the gains obtained by the household next period: for the additional unit saved in the margin, the household gets R_t goods in the next period, which are valued at the expected marginal utility $\mathbb{E}_t[u'(c_{t+1})]$, and discounted by the subjective discount factor β.

Similarly, the left hand side of the Euler equation for stocks shows the marginal cost faced by a household that is buying additional shares of the tree: for each stock, the household must pay a price of q_t, and it has a marginal utility loss of $q_t u'(c_t)$. The factor at the end of the left hand side is the wedge between the market price of stocks of the tree and their collateral value (see Appendix A.2). This wedge is non-zero only when the borrowing constraint is binding, which means that the household values the additional service that their asset holding brings by increasing its borrowing opportunities. In turn, the right hand side is the expected benefit received by the household, which is the resale value of the stock, q_{t+1}, and the dividend, d_{t+1}, as valued by the marginal utility of the household, $u'(c_{t+1})$, and discounted by β.

Alternative specifications of the household's problem, such as Jeanne and Korinek (2010), assume that the household's borrowing is constrained by the aggregate number of stocks in the economy, rather than the household's individual holdings. This eliminates the effect of relaxing the borrowing constraint through an increase of the value of collateral in the Euler equation for stocks (i.e., the wedge between the market and collateral values of the tree). The authors claim that the quantitative results of this alternative formulation do not vary significantly with respect to the problem that we are solving.

In our framework, a sudden stop in external financing arises endogenously as a consequence of the households' borrowing decisions. For high levels of leverage, if the borrowing constraint binds, the households are forced to have a fast reduction of debt, which is only possible through drastic declines in consumption. This causes falls in asset prices by increasing today's marginal utility of consumption and discounting more heavily future cash flows. In turn, this reduces the value of collateral, which further tightens the borrowing constraint, and induces more deleveraging. The feedback between asset price reductions, forced deleveraging, and consumption drops, follows ad infinitum, generating a sudden reversal of the capital flows into the country.

Korinek and Mendoza (2013) highlight that when the external borrowing rate is lower than the households' discount factor, the households face a fundamental tradeoff between impatience and insurance. They have an incentive to borrow from overseas in order to consume in advance because interest rates are low. Nonetheless, for high levels of borrowing, a sudden stop is more likely to happen and, given that it is accompanied by a drastic decline in consumption, households have the incentive to save out of the sudden stops region. In the next section, we illustrate the interaction between these two motives using a numerical solution to our model.

2.3. **Constrained efficient allocation.** The fact that the aggregate level of debt determines asset prices, and this in turn affects the borrowing capacity of the households, creates a pecuniary externality in the economy. Individual households do not internalize the effect of their indebtedness on the borrowing possibilities of the rest of the households, which results in Pareto inefficient allocations. In this section, we study the problem of a social planner that internalizes the effect of external indebtedness on the value of collateral and, hence, on the borrowing capacity of the country. In particular, we consider a social planner that can only choose the level of aggregate debt, subject to the economy's borrowing constraint. The planner cannot intervene directly in the trading of the asset that takes place between households, so it tries to affect the equilibrium value of collateral indirectly, by altering the economy's borrowing decisions. We assume that the planner cannot commit to future policies, and we solve for the constrained efficient allocation that he would implement through time-consistent policies.

We follow Klein et al. (2005) in laying out the social planner's problem and in finding its time-consistent solution. In particular, we restrict attention to the case in which policy rules only depend on the current state variables of the economy. This restriction implies that the policy rule of the planner is given by a simple function of the current states, (B, X), that maps them into levels of aggregate bonds, $B' = \Psi(B, X)$. In Appendix A.3, we show that the problem that is being solved by the social planner can be stated as follows. Given an arbitrary future policy rule, $\Psi(B, X)$, and the associated asset pricing function, $\mathcal{Q}(B, X)$, the social planner chooses c and B' that solves the following Bellman equation:

$$W(B, X) = \max_{c, B'} \left\{ u(c) + \beta \mathbb{E}\left[W\left(B', X'\right) | X \right] \right\}$$

subject to

$$\begin{aligned} c + \frac{B'}{R(X)} &= d(X) + B, \\ -\frac{B'}{R(X)} &\leq \kappa \bar{\mathcal{Q}}(B, B', X), \end{aligned}$$

and the valuation of collateral is consistent with the household's trading of the stocks of the tree:

$$\bar{\mathcal{Q}}\left(B, B', X\right) = \beta \mathbb{E}\left[\left. \frac{u'\left(d(X') + B' - \frac{\Psi(B', X')}{R(X')}\right)\left[\mathcal{Q}(B', X') + d(X')\right]}{u'\left(d(X) + B - \frac{B'}{R(X)}\right)} \right| X \right]. \tag{5}$$

In the appendix we prove that this is the relevant equilibrium pricing condition that the planner faces, given the microeconomic foundations that give rise to our collateral constraint.[8]

Different authors have defined the planner's problem in alternative ways. Bianchi and Mendoza (2011) use the competitive equilibrium price schedule $\mathcal{Q}(B, X)$ and do not allow it to satisfy the frictionless asset pricing condition of the households. They call this the problem of the "financial regulator", and use it to argue that sudden stops are preceded by overborrowing, and there is a role for policy to improve upon competitive equilibrium allocations by reducing external borrowing. Our description of the planner's problem is essentially the same as

[8]Following the literature on optimal taxation under commitment, this condition has been referred to as an implementability constraint.

Bianchi and Mendoza (2013) because we also study time-consistent policies in which the planner affects the value of collateral only through the choice of indebtedness, but households update their valuation of the tree consistent with the planner's policies.

The planner's decision now internalizes the fact that increasing households' savings affects equilibrium asset prices, which in turn alters the value of collateral in the borrowing constraint. In particular, the functions that solve the planner's problem, $c = \hat{\mathcal{C}}(B, X)$ and $B' = \hat{\Psi}(B, X)$, must satisfy the following condition:[9]

$$u'(\hat{\mathcal{C}}(B,X)) - \mu(B,X)\left[1 + \kappa R(X)\xi(B,X)\right] = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\mu(B',X')\psi(B',X')\right], \tag{6}$$

where

$$\psi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B}, \qquad \xi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B'},$$

and $\mathcal{C}(B,X) = B + d(X) - \frac{\Psi(B,X)}{R(X)}$.

In order to gain some intuition on how the planner internalizes the pecuniary externality, let us first focus on the case in which the collateral constraint is not binding in the current period, $\mu(B,X) = 0$. In this case, equation (6) becomes:

$$u'(\hat{\mathcal{C}}(B,X)) = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) - \kappa\mu(B',X')\psi(B',X')\right].$$

The planner's intervention considers not only the possibility of a binding borrowing constraint and how tight it is through the $\mu(B',X')$ term, but also the risk associated with the size of the price externality through the $\kappa\psi(B',X')$ term. Conditional on today's collateral constraint being non-binding, if the future price schedule were constant with respect to debt, the planner would not intervene, regardless of the possibility of the borrowing constraint being binding. Likewise, if there were an externality from borrowing but the planner did not expect the borrowing constraint to bind in the following period, he would not have a reason to distort the households' borrowing decisions. In the appendix, we show that:

$$\psi(B,X) = -\frac{u''(\mathcal{C}(B,X))}{u'(\mathcal{C}(B,X))}\mathcal{Q}(B,X), \tag{7}$$

which implies that the price externality depends on the level of asset prices and the coefficient of absolute risk aversion of the representative household.[10]

Let us now consider the case in which the collateral constraint is binding in the current period. In this case, $\mu(B,X) > 0$, and equation (6) now includes an additional term related to a partial derivative of an unknown function, $\bar{\mathcal{Q}}$. Notice that this is the relevant case in which a time inconsistency problem arises for the planner. The term $\xi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,B',X)}{\partial B'}$ shows that if the borrowing constraint is currently binding, the planner has an incentive to

[9] Klein et al. (2005) call this a "generalized Euler equation" because it is a functional equation of an unknown equilibrium object, in this case $\bar{\mathcal{Q}}$.

[10] The fact that $\psi(B,X)$ can be written in terms of unknown functions, rather than partial derivatives of unknown functions simplifies the analysis of the functional equation.

affect current asset prices by making future promises that would not be time consistent for a committed planner.[11] In the problem of the planner, we assumed that an arbitrary future policy rule, $\Psi(B, X)$, and its implied asset pricing function, $\mathcal{Q}(B, X)$, are take as given. Hence, the current planner can only affect the pricing function by choosing B' and then having the future planner make his decision based on $\Psi(B', X')$, rather than committing to B' and B''. In Appendix A.3, we provide an expression for $\xi(B, X)$ that shows explicitly how it relates to the planner taking future policy rules as given.

Given the characteristics of the social planner's problem, it is straightforward to define a recursive constrained efficient allocation, conditional on arbitrary future planners' policy rules. Our definition of a constrained efficient allocation further requires that the these policy rules be time consistent. In other words, we require that the policy that solves the strategic game being played by sequential planners is a fixed point, deriving in a Markov stationary policy rule. We provide further details and formal definitions of these concepts in the appendix.

3. The dynamics of sudden stops, optimal capital flow management, and external interest rates

3.1. Parameterization and numerical solution. In order to illustrate the general equilibrium interaction of the borrowing constraint and the external shocks, we present a numerical solution of the model. We use a utility function from the constant relative risk aversion family:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Table 1 presents the baseline parameterization of the model for an annual time frequency. The parameters for preferences are standard in the literature of small open economies. Our choice of the relative risk aversion, $\gamma = 2$, lies in the lower end of the values used for emerging economies in the open economy business cycle literature. Hence, the quantitative effects of volatility on real allocations and asset prices that we show are, in principle, conservative. The mean of the dividends process, d, is normalized to one, so we can easily interpret the measurements of consumption, savings, and asset prices relative to the mean annual income. The parameter of the collateral constraint, $\kappa = 0.04$, is chosen to match the ratio of foreign liabilities to GDP observed in a sample of emerging markets in the 1990-2011 period, which averaged 66.7%.[12] In the model, the ergodic mean of the debt-to-output ratio is 65.6%.

We estimate the parameters that rule the regime-switching VAR given by (1) for a group of emerging markets using the maximum likelihood approach of Reyes-Heroles and Tenorio

[11]See Bianchi and Mendoza (2013) for a detailed explanation of the difference between a planner with and without commitment.

[12]This is calculated using data from the updated and extended External Wealth of Nations database of Lane and Milesi-Ferretti (2007). The figure corresponds to the countries in Sample 1 described in Reyes-Heroles and Tenorio (2015). As a reference, an alternative calibration target could have been the average net foreign asset to GDP ratio, which amounts to 27.8% of GDP in our sample.

Table 1. Baseline parameterization

Parameter		Value	Target
Time discount	β	0.96	Standard value
Relative risk aversion	γ	2	Standard value
Dividends	d	1	Normalization
Collateral constraint	κ	0.04	Debt-to-output ratio

(2015), with the data corresponding to Sample 1. The only difference with respect to our estimations in the referred paper is that here we use annual data, which better corresponds to the timing of our model. Quarterly GDP figures were annualized and then log-linearly detrended, and monthly interest rate data was averaged arithmetically. The estimated process is:

$$\begin{pmatrix} z_t \\ r_t \end{pmatrix} = \begin{pmatrix} 0.0052 \\ 0.0025 \end{pmatrix} + \begin{pmatrix} 0.6079 & -0.1321 \\ 0.1289 & 0.8261 \end{pmatrix} \begin{pmatrix} z_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^r \end{pmatrix}, \tag{8}$$

and the covariance and transition matrices are composed of:

$$\begin{aligned} \sigma^z &= 0.0312, \quad \rho = -0.4048, \quad \pi_L = 0.9610, \\ \sigma_L^r &= 0.0150, \quad \sigma_H^r = 0.0661, \quad \pi_H = 0.7468. \end{aligned}$$

This features similar properties to the models estimated at a monthly frequency in Reyes-Heroles and Tenorio (2015). A further description of the business cycle implications is provided in that paper.

The ergodic mean of the output and the interest rate processes can be obtained by inverting the VAR as follows:

$$\mathbb{E} \begin{pmatrix} z_t \\ r_t \end{pmatrix} = (\mathbf{I} - \hat{A}_1)^{-1} \hat{A}_0 = \begin{pmatrix} 0.0066 \\ 0.0196 \end{pmatrix},$$

where $\hat{A}_0$ and $\hat{A}_1$ denote the estimated matrices in (8). The long-run average of the external interest rate is, thus, 1.96%, which is considerably below the households' discount rate of $(\beta^{-1} - 1) \approx 4\%$. This gives the households an incentive to borrow from the exterior in order to consume upfront.

The two regimes of the VAR have considerably different interest rate volatilities. In the low volatility regime, the standard deviation of interest rate shocks is small, $\sigma_L^r = 1.50\%$, leading to a very low refinancing risk for bond holdings. In contrast, in the high volatility state, the standard deviation is 4.4 times higher, $\sigma_H^r = 6.61\%$, which induces a large uncertainty in the future access to debt financing for the economy. The transition matrix between the two volatility states has a high persistence: the mean duration of low and high volatility episodes is 25.6 and 3.9 years, respectively. In the long run, the system spends 86.6% of the time in the low volatility state.

Our estimates for the variance of the external interest rate are consistent with the findings in the literature (e.g., Fernández-Villaverde et al., 2011). A limitation in our estimated process is that the shocks to the interest rate are symmetric: when volatility increases, it

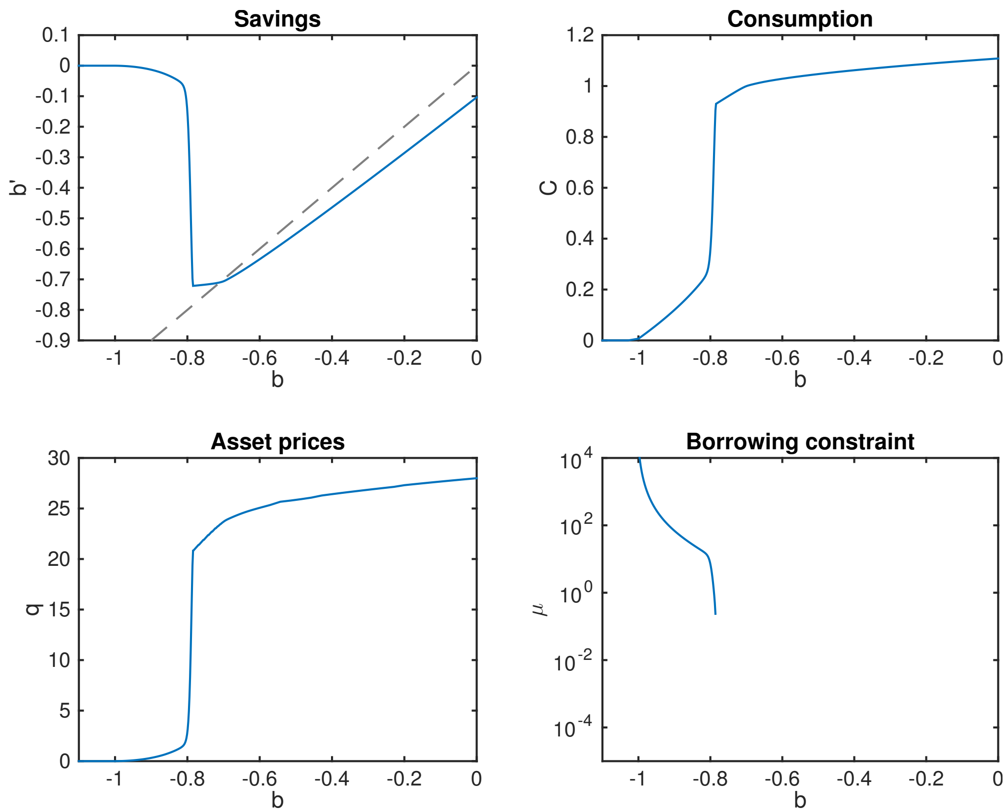

Figure 1. Recursive competitive equilibrium: savings rule, consumption, asset prices, and multiplier on the borrowing constraint.

is equally likely for it to reach high deviations above or below the mean. We opt not to introduce asymmetries in our estimation for the sake of parsimony and simplicity. However, an estimation of the VAR model with additional degrees of freedom can be conducted to assess the quantitative relevance of asymmetric shocks.

We use a global solution method to characterize the recursive competitive equilibrium of the economy in a discretized version of the aggregate state space. We use a grid of 300 points for household savings, placing 80% of them around the region where the borrowing constraint binds, in order to better capture the nonlinearities of the model. We discretize the estimated VAR process using a two-dimensional variation to the Tauchen (1986) method that allows for different levels of variance of the shocks. We use a grid of 7 points for output shocks and 15 points for the interest rate, to better capture the effects of changing volatility of the latter variable. We truncate the grids in order to include 95% of the probability mass of shocks at the ergodic distribution, which was approximated by simulating the VAR for one million periods. To solve the system of rational expectations with occasionally binding constraints, we use an adaptation of the endogenous grid method of Carroll (2006). Appendix B describes in detail our algorithm and its numerical accuracy.

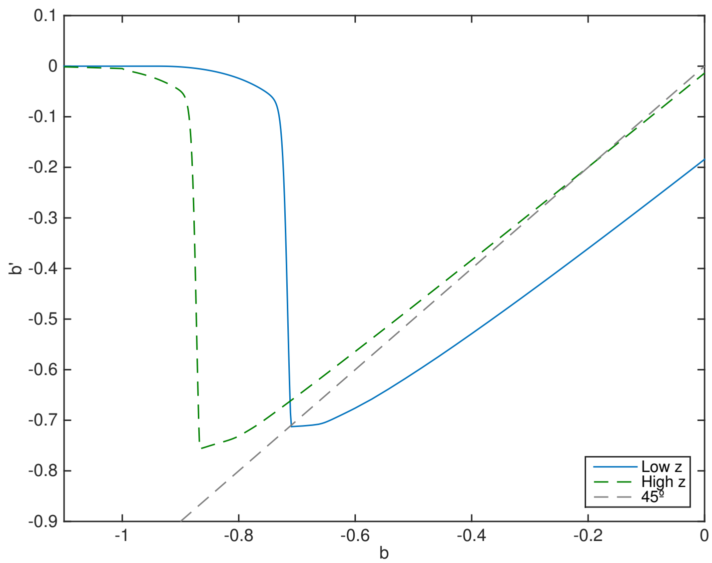

Figure 2. Savings rule: different endowment levels

3.2. **Description of the competitive equilibrium.** Figure 1 depicts the numerical solution to the recursive competitive equilibrium. In the first panel, we show the representative household's savings rule $\mathcal{B}(B, X)$ as a function of the initial level of aggregate savings B. This decision rule is non-monotonic: for high levels of wealth, the savings rule is upward sloping, as expected. Given that the average interest rate is below the households' discount factor, there is an incentive to increase the economy's indebtedness, which is reflected on the fact that the savings rule lies below the 45 degree line. If the amount of debt reaches high enough levels, the borrowing constraint becomes binding. In this situation, the households must reduce their consumption in order to lower their stock of debt, as displayed in the second panel of the figure. This causes an increase in the marginal utility of contemporaneous consumption, which in turn induces a higher discount of future cash flows and a consequent drop in asset prices. This is shown in the third panel, which depicts the equilibrium asset prices, $\mathcal{Q}(B, X)$, as a function of savings B. The sharp drop in the value of collateral forces a large deleveraging, as shown in the first panel, which feeds back into further consumption cuts and asset price falls, ad infinitum.

Episodes with binding borrowing constraints in our economy are accompanied with sharp declines in consumption, and since the households are inelastic in terms of intertemporal substitution, this fast deleveraging entails high utility losses. Therefore, households have a precautionary savings motive around the region in which borrowing constraints bind. The first panel of Figure 1 shows that the rate of indebtedness is lower around this region: the slope of the savings rule slowly decreases as the level of debt increases, before hitting the borrowing constraint. Hence, the precautionary motive gains an increasing importance vis-à-vis the impatience motive in the households' problem.

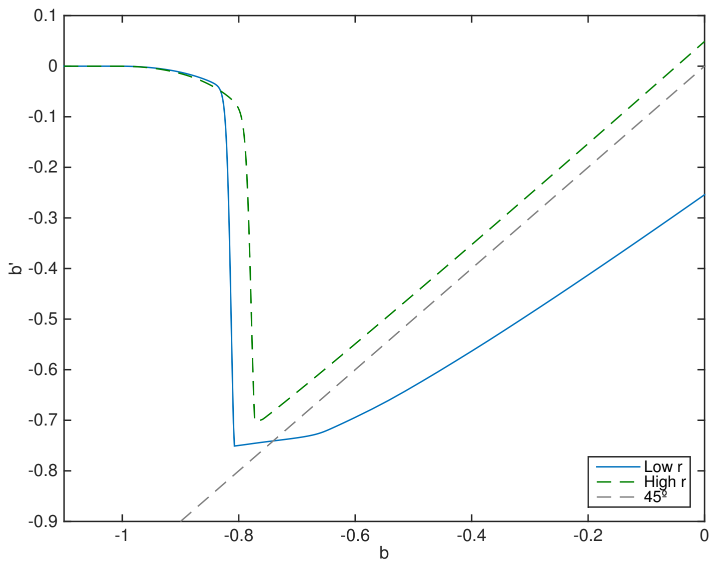

Figure 3. Savings rule: different interest rates

In Figure 2, we compare the savings decision rule for two different levels of the contemporaneous endowment. When there is a high level of output in the period (green dashed line), there tend to be greater savings from the households in the region where borrowing constraints do not bind. This occurs because households wish to smooth consumption across time, and since the process for the endowment is mean reverting, it is likely that in future periods there will be a lower output than in the present. However, since the endowment process is persistent, a high level of contemporaneous output predicts high levels of output in the near future, which in turn increases the value of the Lucas tree for the household. This causes an increase in the value of the collateral available in the economy, which raises the borrowing capacity of the households. Hence, the borrowing constraint starts binding at higher levels of debt, as the green dashed line shows.

Figure 3 compares the savings decision rules for two different levels of the external interest rate. Away from the borrowing constraint, the interest rate has the usual impact on the economy: when the country faces a higher cost of borrowing (green dashed line), it tends to increase its savings. However, in the vicinity of the borrowing constraint, changes in the interest rate have an additional effect: an increase in the interest rate causes a decline in the stochastic discount factor (in expectation), which in turn reduces the value of the tree because its future flows are discounted more heavily. Hence, when the country faces higher interest rates, the value of collateral is lower, and the borrowing constraint starts binding for lower levels of debt.

In Figure 4 we compare the decision rules in the economy for two different levels of the variance of the external interest rate, σ^r. We keep constant the level of interest rates, but only compare the decision rules for the two levels of such variance. The figure shows that the savings rule for the high volatility state lies slightly above the one for low volatility. This

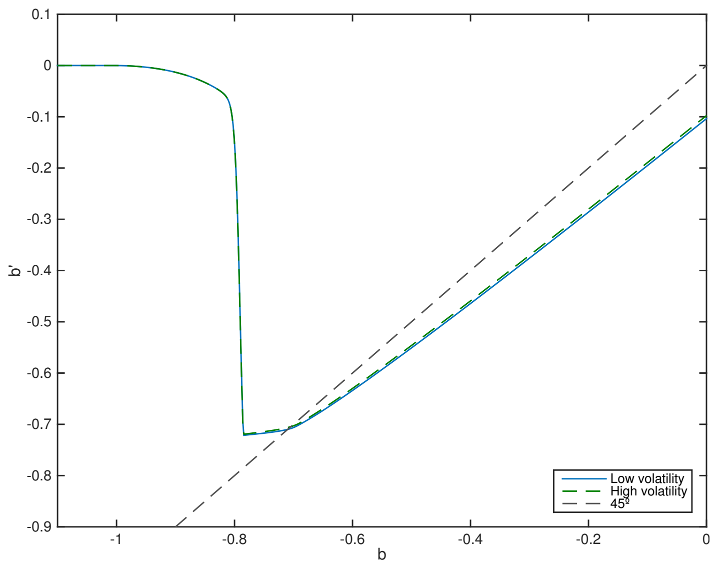

Figure 4. Savings rule: different levels for the variance of the interest rate

was the expected outcome, because the households should have a higher precautionary saving motive when they face a world with higher uncertainty. Nonetheless, the magnitude of the difference between both decision rules is considerably small, so these shocks do not modify the household's saving substantially.

In our model, the small effect of external volatility on equilibrium allocations arises from the absence of a complete production economy with capital accumulation. Fernández-Villaverde et al. (2011) provide, to our knowledge, the first solution of an open economy business cycle model that faces shocks to the volatility of the external interest rates. In their model, there is a significant response of the economy to these shocks due mainly to a capital accumulation motive. The mechanism that they highlight is the importance of external debt as a hedge for domestic income shocks: in the real business cycle framework, most of the risk in the households' consumption arises from shocks to the domestic productivity level. The external locally-risk-free debt is a good hedge for domestic risk arising from productivity fluctuations. However, when the rollover risk of external debt increases, foreign bonds are less useful as a hedge, which implies that the economy must cut on their holdings of capital to reduce their exposure to domestic risk. As they do so, they cause a decrease in output in the subsequent periods, which reduces the wealth of the economy, and induces a reduction in consumption and foreign indebtedness.

Unlike Fernández-Villaverde et al. (2011), we do not incorporate capital accumulation and production in our framework, since we are interested in isolating the policy response of a planner that only cares about the incidence and consequences of binding borrowing constraints along the business cycle. By introducing the production mechanism, we would potentially be increasing the planner's incentives to engage in ex ante and ex post interventions to reduce the incidence of crises and the size of their effects. In this sense, our exercise is conservative

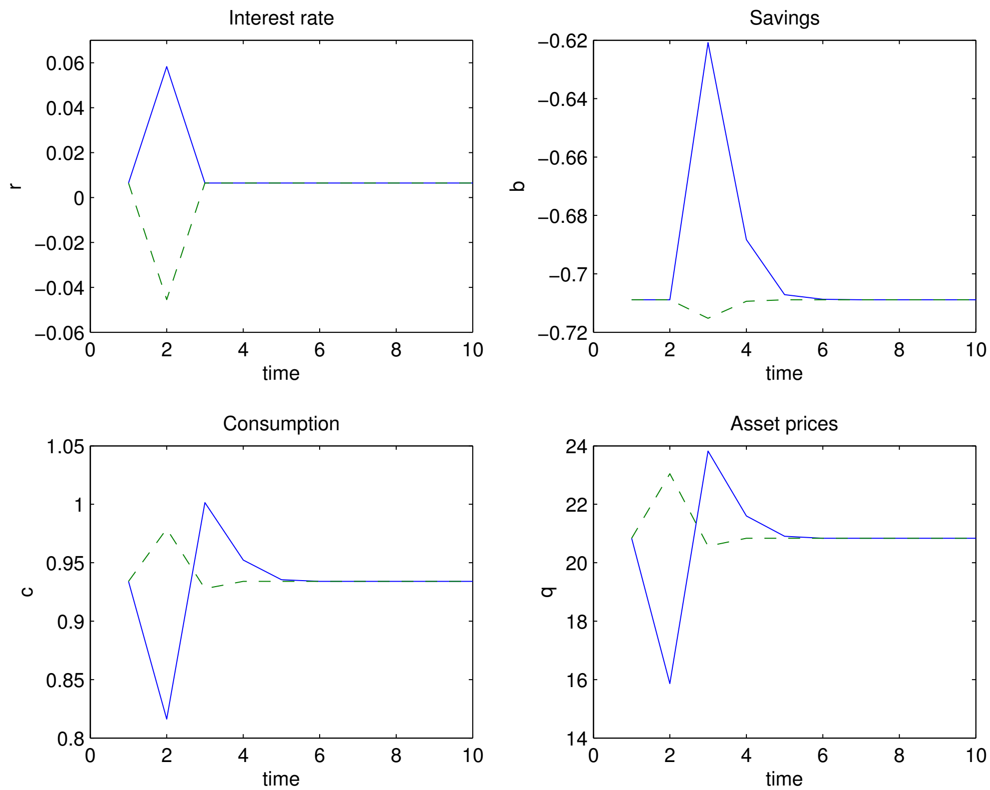

Figure 5. Impulse-response functions

when it comes to the reasons that a social planner would have to prevent the occurrence of binding borrowing constraint episodes.

We now turn to describe the nonlinear dynamics of the economy. In Figure 5 we show our simulated impulse-responses around the steady state where the economy would remain if the level of output from the tree remained permanently constant at 2 standard deviations below its mean, the interest rate remained at 0.6%, and the variance of the interest rate remained permanently at 6.6%, i.e., in the high volatility regime. We then give a ±5.2% shock to the interest rate for one period, and bring the interest rate to 0.6% thereafter. First, we explain the effects of the interest rate decrease on the rest of the economy (green dashed line). The immediate effect is an incentive for the households to consume in advance. Therefore, they increase their consumption by 4.8% on the first period, without significant changes in the net savings of the economy. Asset prices show a 10.6% increase in the first period because the households are discounting future cash flows less, but they revert close to their long-run level in the following period.

In contrast, the economy responds very differently to an increase in the interest rate of the same magnitude. The immediate effect of the shock is a decline in asset prices, as shown in the last panel of the figure (blue solid line). The decline in the value of collateral causes the borrowing constraint to bind for a period, which forces a reduction in consumption in order

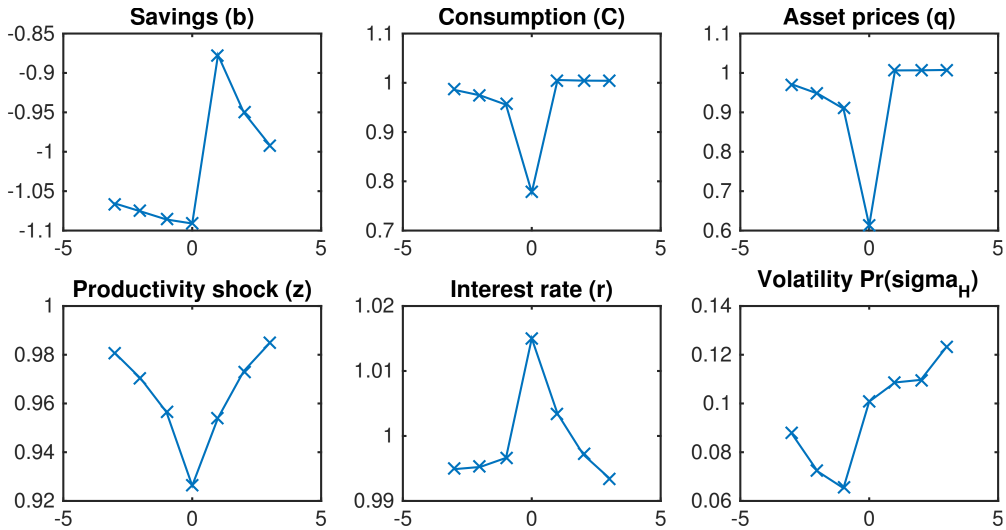

Figure 6. Event studies of binding borrowing constraints in competitive equilibrium

to cut off the level of debt. As mentioned before, the feedback between deleveraging and the decline of asset prices amplifies the initial shock: consumption initially falls by 12.6%, and asset prices drop by 23.9%. This carries a sharp reduction in foreign debt: it goes from 70.9% of average output to 62.1% in just one period. In addition, as the graphs show, the sudden deleveraging has long-lasting effects: given that there is a lower level of debt, asset prices remain high because there is a low probability of hitting the borrowing constraint again in the near future. Moreover, since the country has accumulated more savings, the household increases its consumption in the subsequent periods because it remains relatively impatient with respect to the rest of the world, until the stock of debt converges back to its long-run level. This exercise exemplifies the nonlinear and asymmetric dynamics of the model that arise from the presence of an occasionally binding borrowing constraint.

We simulate the model for one hundred thousand periods to study the prevalence of binding borrowing constraints and their effects around these events. We find that in our baseline parameterization, a binding borrowing constraint is a rare event: it only takes place in 1.82% of the periods. Even in the periods preceding the actual occurrence of a binding constraint, the model assigns conditional probabilities to this event below 10% on average.

In Figure 6, we present event studies by averaging the equilibrium variables around the period in which the borrowing constraint binds. All the variables are divided by their average value in "normal times", i.e., in periods in which the borrowing constraint is non-binding. The only exception is the window for interest rate volatility, which shows the fraction of episodes in which the high volatility regime is prevailing. Each panel shows the normalized average of the variable from $t-3$ to $t+3$, where t is the moment in which the borrowing constraint binds. In the first panel, we see that binding constraints arise from periods in which the economy has a relatively large stock of debt: the average level of debt before sudden stop periods is almost 10% higher to the average debt in non-binding periods. In the panels of the second row, we can see that binding borrowing constraints are typically accompanied by low levels of the endowment, z, and drastic increases in the interest rate, r.

To contrast our model with the empirical evidence, we follow the literature in associating a period in which a borrowing constraint binds in the model with the occurrence of a sudden stop in the data. From this perspective, the prevalence of sudden stops in the model is considerably lower than in the data. Under the definition of sudden stops adopted in Reyes-Heroles and Tenorio (2015), the prevalence of these episodes in the emerging markets studied lies between 14.6% and 15.21% of the periods (measured in months), depending on the sample of countries that is considered.

Nonetheless, the evolution of the modeled economy around sudden stops is consistent with the empirical evidence presented in Reyes-Heroles and Tenorio (2015) regarding the dynamics of the external interest rate. Both in the model and in the data, a sudden stop is associated with a sharp increase in the interest rate: the model predicts that sudden stops happen when the interest rate increases on average 1.5 percentage points with respect to the normal times mean, whereas in the data, the interest rate increases between 1 and 2 percentage points in the 12 months that follow the beginning of such episodes. In addition, the model predicts that sudden stops take place after periods of relatively low interest rate volatility, in the moment in which volatility switches to the high regime, allowing for large upward shocks in the level of the interest rate. Again, this pattern is consistent with the sudden rise in volatility in the year of the sudden stop that we observed in the data.

The fourth panel of Figure 6 shows that a binding borrowing constraint is typically preceded by a sequence of negative output shocks, and an abnormally large negative shock in the period in which the constraint binds, that brings the level of output almost 8% below its normal times level. This contrasts with the empirical evidence in two respects. First, sudden stops are typically preceded by economic expansions, of around 1%, in the sample studied by Reyes-Heroles and Tenorio (2015). Second, the empirical output declines after the episode begins are relatively modest, of around 2% relative to its normal times level. In terms of consumption and asset prices, the dynamics of the model agree with the empirical patterns of balance of payment crises: these are usually accompanied with sharp declines in consumption and asset prices. However, the fall in consumption that arises in our model, of about 20% below the normal times level, is considerably higher than its empirical counterpart, of about 2 or 3% in the countries studied by Korinek and Mendoza (2013).

3.3. **The constrained efficient allocation and optimal capital flow management.** We use the same parameterization of the previous section to characterize quantitatively the solution to the planner's problem. In this section, we follow Jeanne and Korinek (2010) in postulating that the following condition holds:

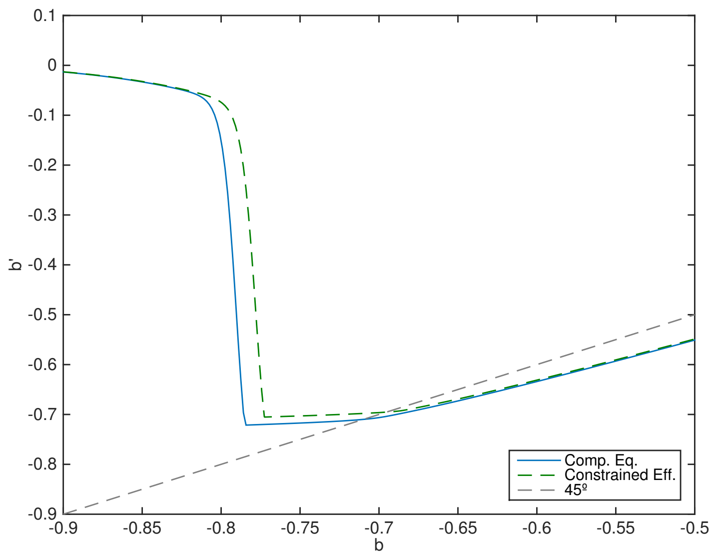

Figure 7. Savings rule: constrained efficient allocation versus competitive equilibrium

Assumption 1. *The parameters and stochastic processes of the economy are such that the equilibrium pricing function satisfies:*

$$1 + \kappa R(X)\xi(B, X) > 0.$$

This condition guarantees that there exists a unique level of future savings in the planner's problem, B', for which the collateral constraints holds with equality. If this condition were not true, it could be the case that an increase in household debt relaxes the constraint by increasing the value of collateral. This is in principle a counterintuitive outcome, but it is possible to have a negative derivative of the $\bar{Q}$ schedule of equation (5) with respect to B', due to the concavity of the utility function (see Appendix A.3 for an expression of $\xi(B, X)$ based on marginal utilities and equilibrium objects). Jeanne and Korinek (2010) prove that under this assumption, the Euler equation for the planner's problem (6) simplifies to:

$$u'(\mathcal{C}(B, X)) - \mu(B, X) = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B', X')) + \kappa\mu(B', X')\psi(B', X')\right].$$

For the remainder of this section, we describe the optimal decision rule of the planner, and the associated equilibrium outcomes, based on this version of the Euler equation.

Figure 7 compares the savings rules for the households in the competitive equilibrium and the solution to the planner's problem. As Bianchi and Mendoza (2011) have previously noted, the savings rule in both problems are similar in most of the state space, but they differ considerably in what they call the "high externality region", where the borrowing constraint has a high probability of binding, and the asset price schedule becomes steeper as a function of savings.

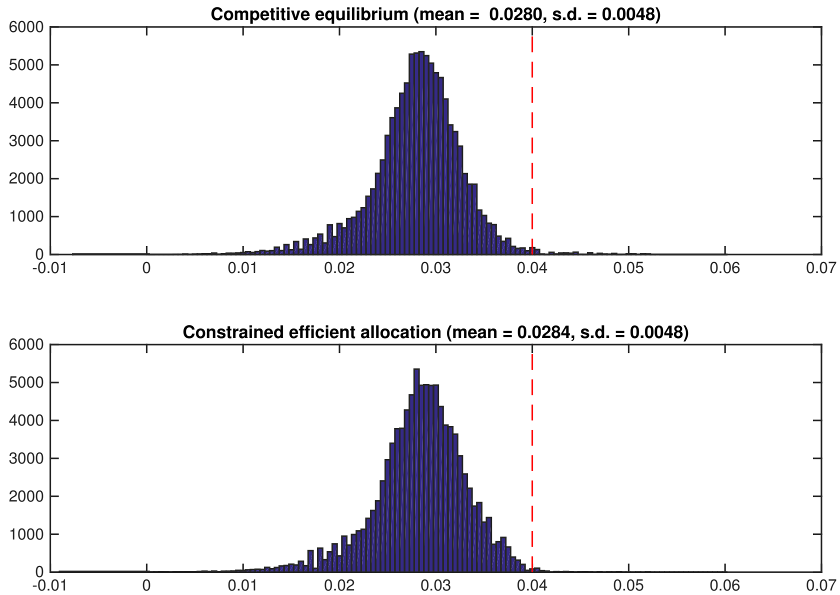

Figure 8. Histograms of leverage

Even though the savings rules do not show large differences between the competitive equilibrium and the planner's problem, there are indeed some differences in the dynamics of both problems. First, we find that the planner is able to reduce the frequency of sudden stops from 1.82% of the periods in the competitive equilibrium, to 1.61% in the constrained efficient allocation. However, as we observe in Figure 8, the amount of leverage in the planner's economy does not change considerably with respect to the competitive equilibrium. Here, we define leverage as the discount value of debt divided by the market value of the Lucas tree, $-b_{t+1}/R_t q_t$. The red line marks the level of leverage where the borrowing constraint binds, given by $\kappa = 0.04$ in our numerical example. Both histograms of leverage have a similar mean of around 0.028 and the same dispersion, of 0.0048.

Nevertheless, the planner's actions do have an effect in the severity of the sudden stops that the economy faces. In Figure 9, we show event studies around the periods in which the borrowing constraint binds in the planner's economy. The outcomes corresponding to the planner's problem are depicted in green dashed lines. We observe that the consequences of a binding borrowing constraint are considerably milder in the planner's allocation, compared to the laissez faire competitive equilibrium: consumption decreases by less, asset prices remain higher, and the deleveraging is slower. Even though the average decline in the endowment is roughly the same in both economies, it takes a larger interest rate shock to hit a borrowing constraint in the constrained efficient economy. This is accompanied by a sudden increase in volatility, that enables the interest rate shock to reach high realizations.

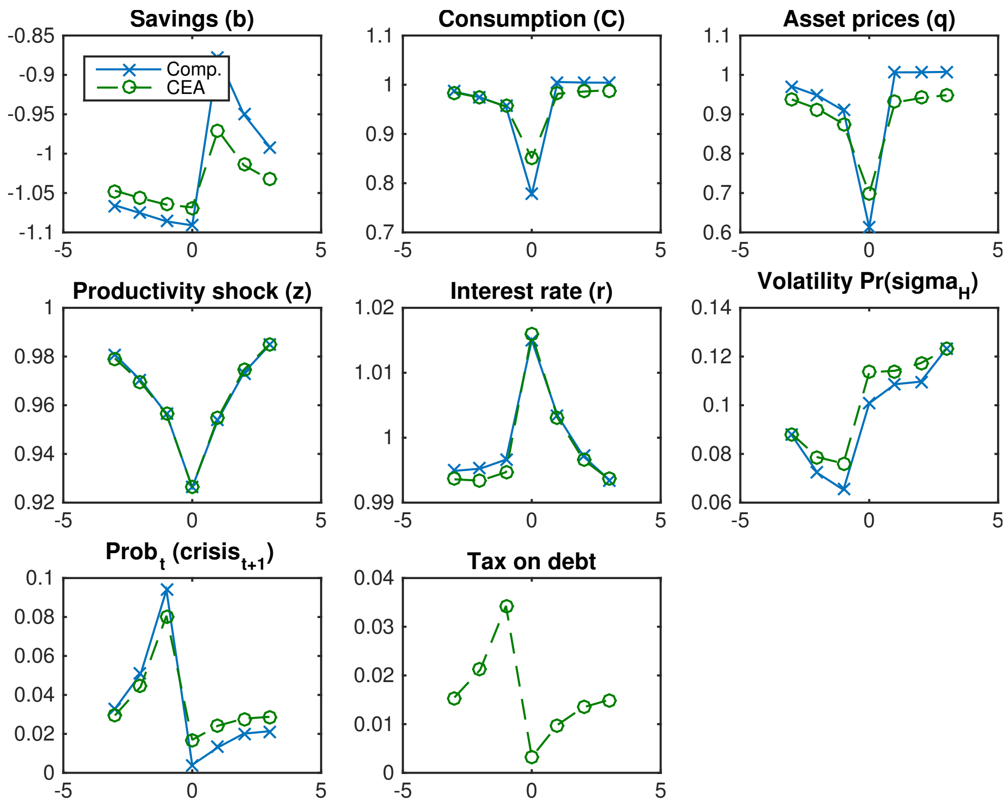

Figure 9. Simulated sudden stops in constrained efficient allocation

3.3.1. *Decentralization.* We now explore how the planner responds to the exogenous shocks that the economy faces. To do so, we use the fact pointed out by Jeanne and Korinek (2010) and Bianchi and Mendoza (2011) that the constrained efficient allocation that solves the planner's problem can be decentralized with a state contingent "macroprudential" tax on debt. These authors show that the wedge on the households' gross interest rate that implements the allocation of the planner's problem is:

$$\tau(B, X) = \frac{\mathbb{E}\left[\kappa\psi(B', X')\mu(B', X')|X\right]}{\mathbb{E}\left[u'(c(B', X'))|X\right]}, \tag{9}$$

where $B' = \Psi(B, X)$ is the optimal level of savings chosen by the planer when initial savings are B, and shocks X are realized. The size of the planner's intervention is, thus, determined by the expected marginal welfare gain of reducing households' indebtedness: the value of reducing households' debt by a unit is equal to the increase in the value of collateral, $\kappa\psi(B', X')$, times the marginal value of relaxing the collateral constraint, $\mu(B', X')$.

In Figure 10 we depict the optimal tax on debt, $\tau(B, X)$, as a function of the initial savings of the country, B, for two different levels of the endowment shock. Focus first on the green dashed line, corresponding to a high realization of the endowment. For high levels of household savings (to the right of the graph), the borrowing constraint is less likely to bind, which makes the planner's intervention small or even null. Then, as debt starts accumulating,

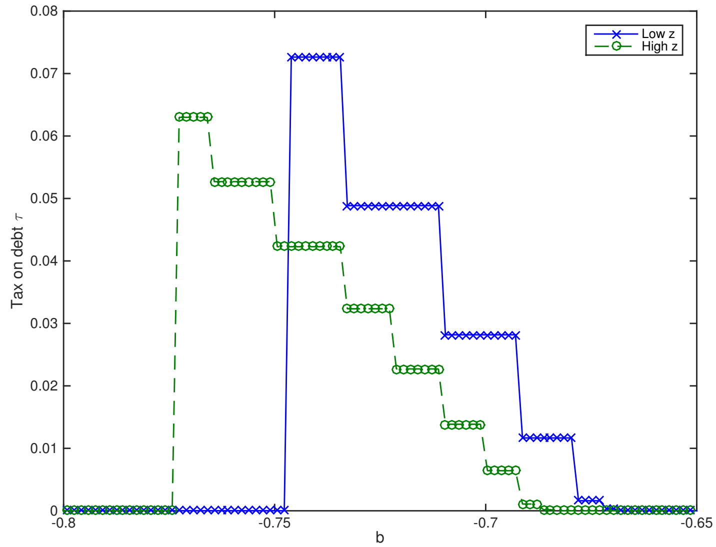

Figure 10. Tax on debt as a function of savings: different endowment levels

two things happen: (i) the borrowing constraint is more likely to bind, and it becomes more stringent, which derives in a higher multiplier $\mu(B', X')$, and (ii) the size of the pecuniary externality $\psi(B', X')$ is higher because consumption is lower. Both of these effects call for a larger intervention by the planner, reflected in a higher macroprudential tax. In our numerical example, the tax rate amounts to a few percentage points over the gross interest rate, which considerably increases the after-tax interest rate paid by the households. In the figure, we also see that for higher levels of debt, the borrowing constraint binds and the households are forced to delever drastically by the price-debt mechanisms of the model. This brings the stock of debt away from the borrowing constraint for the immediate future. In this case, the tax on debt is zero because the economy is not borrowing-constrained in the upcoming period. Thus, this model has no space for ex post intervention; the planner's actions to eliminate pecuniary externalities are only necessary before a borrowing constraint binds.

Figure 10 also shows that the macroprudential intervention is always non-negative. The planner taxes debt whenever he expects that reducing households' borrowing has a positive effect on welfare through the internalization of the price effect of debt. From equation (7), we see that the pecuniary externality of debt is always non-negative because the utility function is strictly increasing and concave, and asset prices are non-negative throughout the state-space. On the other hand, the effect of relaxing the collateral constraint is non-negative, because it necessarily increases welfare when the constraint binds, and has a null effect otherwise. The tax on debt is thus given by the expected product of two non-negative random variables, so it must itself be non-negative.

We now analyze how the planner's intervention responds to endowment shocks. The solid blue line in Figure 10 depicts the optimal tax on debt as a function of households' savings for a low realization of the endowment shock, z. In the region where borrowing is unconstrained,

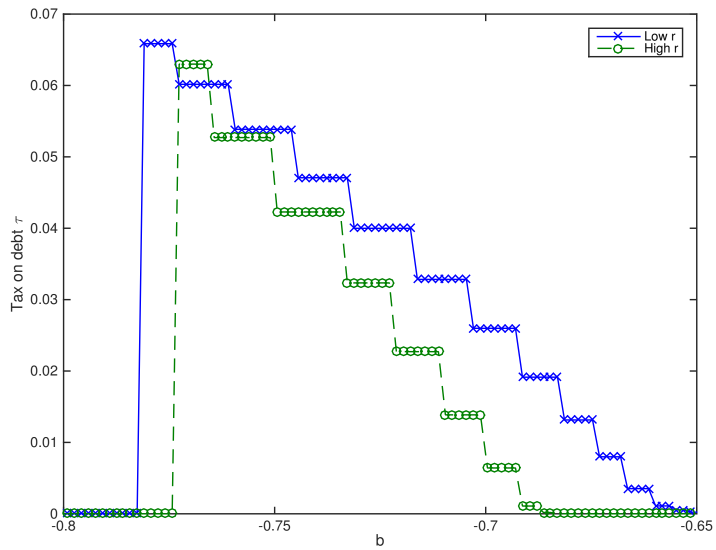

Figure 11. Tax on debt as a function of savings: different interest rates

the tax on debt is typically higher for lower realizations of the endowment, which is explained by the fact that low levels of dividend reduce the value of the Lucas tree, which in turn decreases the value of collateral available, and increases the probability of a binding borrowing constraint in the near future. In addition, the same reasoning explains why the planner's intervention becomes null for lower levels of debt, compared with the intervention for high endowment realizations.

In Figure 11, we describe the dependence of the optimal macroprudential tax on interest rate shocks. The first thing we observe is that the macroprudential tax is almost uniformly lower for high levels of the interest rate, which is consistent with the findings of Jeanne and Korinek (2010). The authors make a comparative statics exercise on how the macroprudential tax changes with different values of the external interest rate. In their exercise, the interest rate is a fixed parameter in the planner's problem, and they compare the steady state value of the tax when the value of the endowment is kept constant. The authors find that the steady state level of the macroprudential tax is decreasing with respect to the external interest rate: as the interest rate increases, the planner has a lower need to reduce households' borrowing because they do so themselves as a response of a higher cost of credit. Our analysis, in contrast, studies the response of taxes to interest rate shocks off the steady state. As Figure 11 shows, for some levels of debt, the tax that results after a high interest rate shock is actually larger than the one corresponding to a low interest rate shock, which is explained by the fact that higher interest rates depress asset prices and reduce the value of collateral, which increases the probability of a borrowing constraint binding and calls for a larger intervention.

Next, we study whether an increase in the volatility of the external interest rate calls for a larger intervention of the social planner. Figure 12 depicts the schedule of tax on debt as a function of household savings, for the two different regimes of interest rate variability.

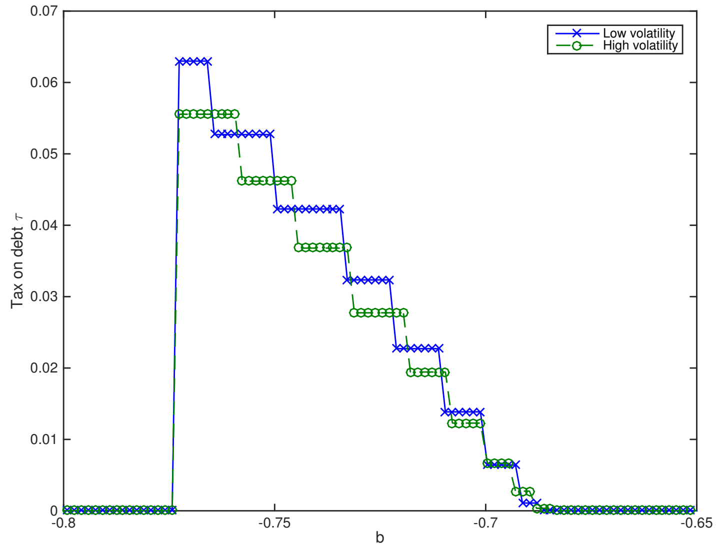

Figure 12. Tax on debt as a function of savings: different variances of the interest rate

We draw two main conclusions from the effect of interest rate volatility on the planner's problem. First, the planner does indeed have a volatility-contingent optimal policy. This contrasts with the result that the savings rules in the competitive equilibrium do not differ considerably between high and low volatility states (see Figure 4). In the constrained efficient allocation, the planner's policy is affected by the volatility of interest rates because, as the variability increases, the economy is more likely to hit states in which the borrowing constraint binds, which usually calls for larger intervention of the social planner.

Our second conclusion is that the size of the optimal planner's intervention is non-monotonic with respect to the volatility of the interest rate. Figure 12 shows that for certain levels of savings, the planner intervenes more when the volatility is high, but in other levels of savings the planner has a smaller intervention. This follows from the fact that the planner is weighing two criteria while choosing the optimal tax on debt: the incidence of sudden stops, and the size of the pecuniary externality. In the following section we show that the interaction between these two factors shapes the response of the planner to volatility shocks.

We now study whether the non-monotonic effect of volatility on taxes is also present in the simulated economy. First, we find that the share of states in which the planner chooses a zero tax on debt is larger when there is high variance than low variance: the planner sets a tax of zero in 59.6% of the periods of high volatility, versus 55.3% of low volatility periods. This is due, partly, to the fact that the economy is more likely to be hit by very low interest rates when the variance is high, and in those states the planner is unlikely to intervene. In Figure 13 we look at the ergodic distribution of the tax on debt, conditioning on low and high volatility states, and ignoring the periods of zero intervention. We see that the positive interventions in the low volatility state have an average of 1.96%, which is larger

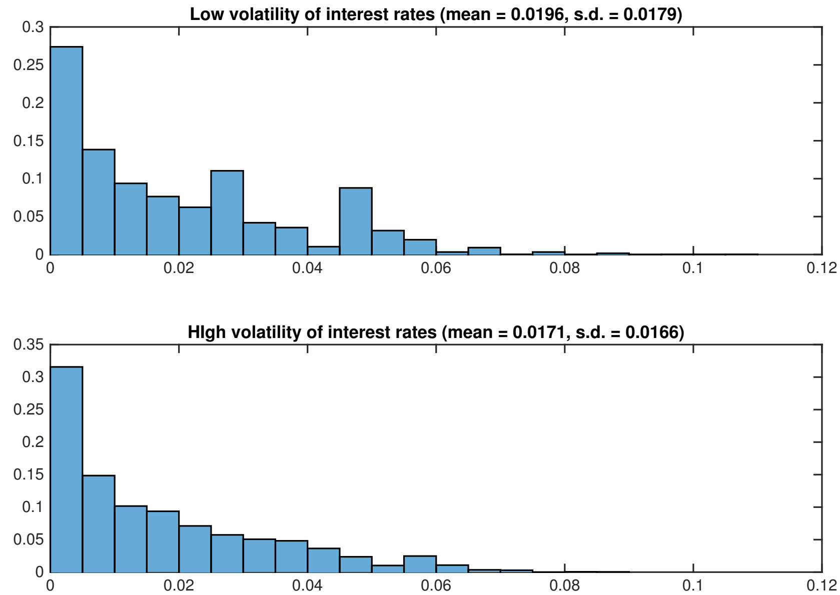

Figure 13. Histograms of tax on debt conditional on variance states

than the average positive intervention in high volatility periods, of 1.71%. Moreover, the highest interventions in our simulations reach 10.7%, and take place only in the low volatility state. In contrast, the highest intervention in the high volatility state is 8.92%.

Finally, we go back to Figure 9 and we observe in the last panel the evolution of the macroprudential tax around the occurrence of a sudden stop. We find that prior to hitting the borrowing constraint, the planner charges on average a tax on debt of around 3.5%, which significantly raises borrowing costs for households, because the average interest rate they face is just 1.96%. Nonetheless, as we previously discussed, the planner does not engage in ex post macroprudential policies: the tax on debt when the borrowing constraint binds is close to zero, given the fact that there is a fast deleveraging taking place that makes it unlikely for a subsequent period to observe a binding borrowing constraint. Therefore, there is no motive for the planner to intervene once the borrowing constraint is already binding.

3.3.2. *Decomposition of the optimal policy.* In this section, we further study the planner's response to the different shocks in the modeled economy. In preliminary numerical exercises, we have found that most of the response of the macroprudential tax to exogenous shocks comes from the numerator of (9), because the denominator remains fairly constant across different states, due to the planner's tendency to smooth households' consumption. Hence, the natural way to proceed is to decompose the numerator of the macroprudential tax in a product term and a covariance term:

$$\mathbb{E}\left[\kappa\psi(B',X')\mu(B',X')\right] = \mathbb{E}\left[\kappa\psi(B',X')\right]\cdot\mathbb{E}\left[\mu(B',X')\right] + \mathrm{Cov}\left(\kappa\psi(B',X'),\mu(B',X')\right),$$

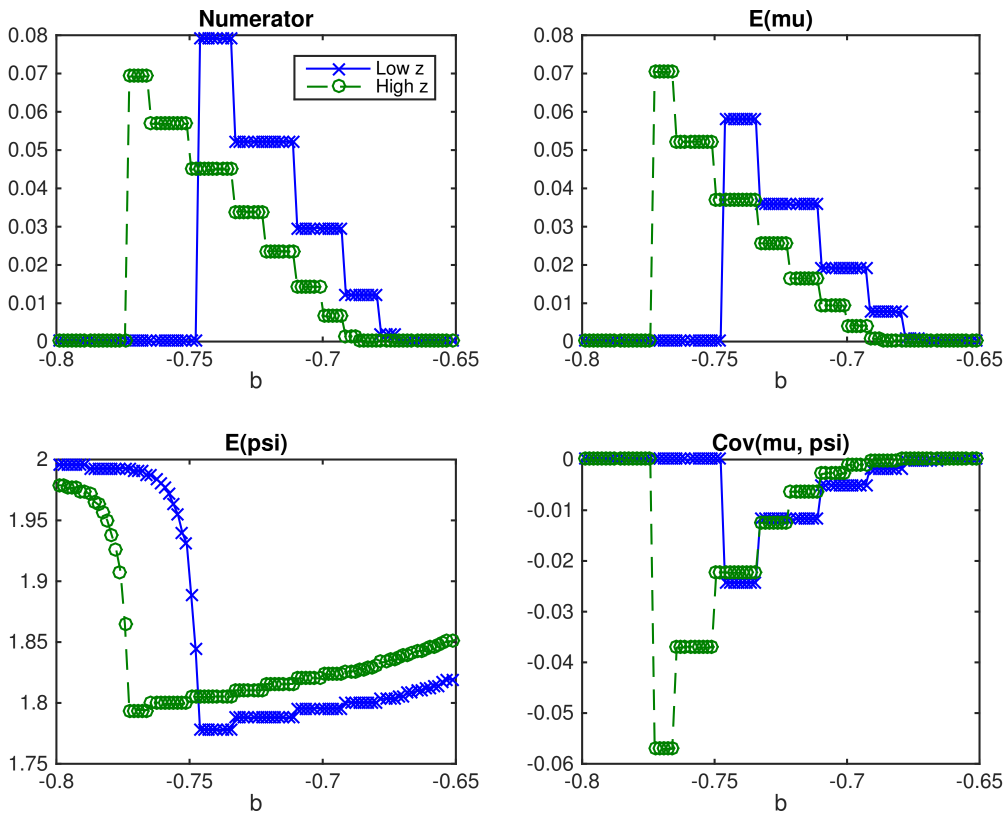

Figure 14. Decomposition of the tax on debt as a function of savings: different endowment levels

where all the moments are conditional on the contemporaneous vector of shocks, X. Thus, the planner's intervention is higher either: (i) when he expects a higher prevalence of binding borrowing constraints and a higher stringency when they bind, through the expectation of the $\mu(B', X')$ term; (ii) when he expects a high degree of pecuniary externalities taking place in the following period, through the expectation of $\kappa\psi(B', X')$ term; or (iii) when he expects these two factors to have a large covariance in the following period.

This last effect is less intuitive, but it has the following rationale: the larger the conditional covariance between the pecuniary externality, $\kappa\psi(B', X')$, and the shadow valuation of the borrowing constraint, $\mu(B', X')$, the larger the planner's efforts will be to reduce the households' borrowing by increasing the tax on debt. This in an effective measure to increase household welfare: if the size of the pecuniary externality were uncorrelated with the shadow valuation of the borrowing constraint, then the planner would not have much ability to improve the households' utility by reducing their borrowing, so his optimal intervention would be small. As the covariance between both criteria increases, the expected welfare effect of the planner's intervention is higher: inducing a reduction in households' borrowing diminishes the pecuniary externality more in the states in which the borrowing constraint is tighter. Thus, the planner finds it optimal to intervene more when these two criteria are correlated.

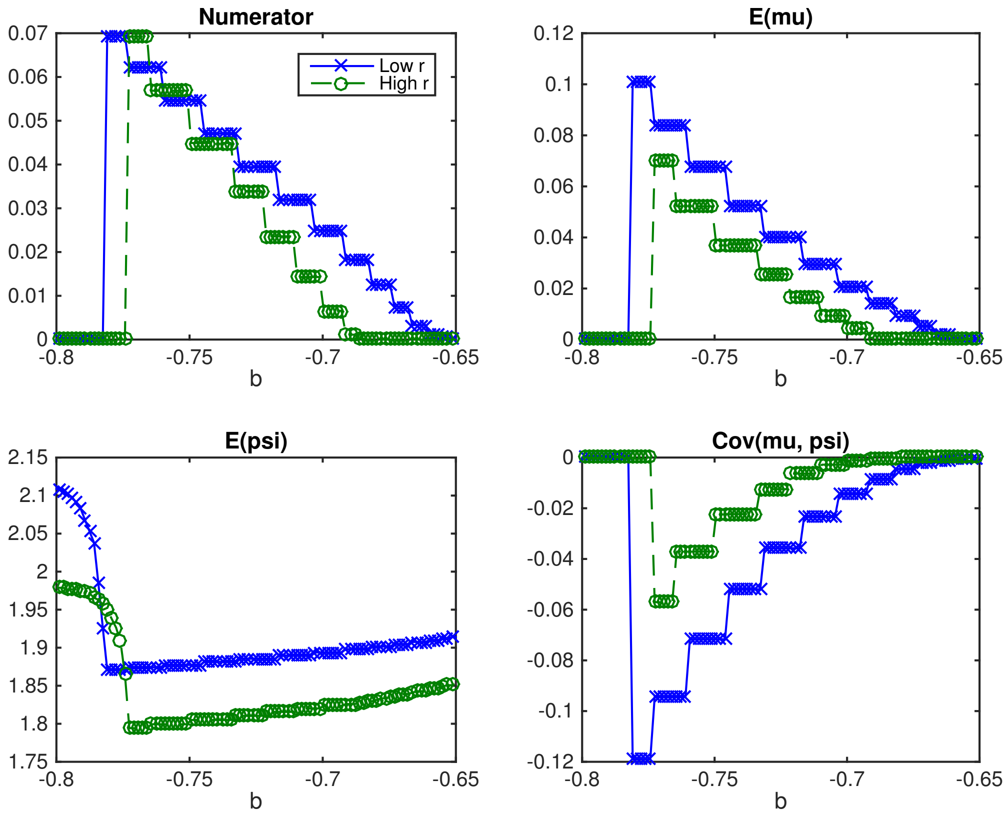

Figure 15. Decomposition of the tax on debt as a function of savings: different interest rates

Figure 14 shows the decomposition of the numerator of $\tau(B, X)$, for two different levels of the endowment shock. The fact that the planner intervenes more after low endowment realizations is driven by the fact that the borrowing constraint is expected to be more stringent in the following period, because current reductions of the endowment cause a decline in asset prices due to the persistence of the shock. The externality effect, ψ, has the opposite direction, since the planner is expecting lower externalities when the realization of the endowment is lower. The covariance effect is negative in this case, but it does not respond significantly to different endowment realizations. Thus, the effect of binding borrowing constraints, μ, is the one driving the increase in macroprudential intervention after low endowment realizations.

In Figure 15 we perform a similar exercise, decomposing the planner's tax on debt for two different levels of the interest rate shock. We wish to explain why the planner's intervention is lower when interest rates increase. From the figure, we learn that both the stringency of future borrowing constraints, and future externalities are expected to be lower when the interest rate rises. On one hand, the increase in interest rates decreases asset prices, which directly lowers the derivative ψ, as can be seen in expression (7). On the other hand, the reduction of interest rates relaxes the left hand side of the borrowing constraint, which reduces its multiplier. The covariance between these two effects is higher (or less negative) when the

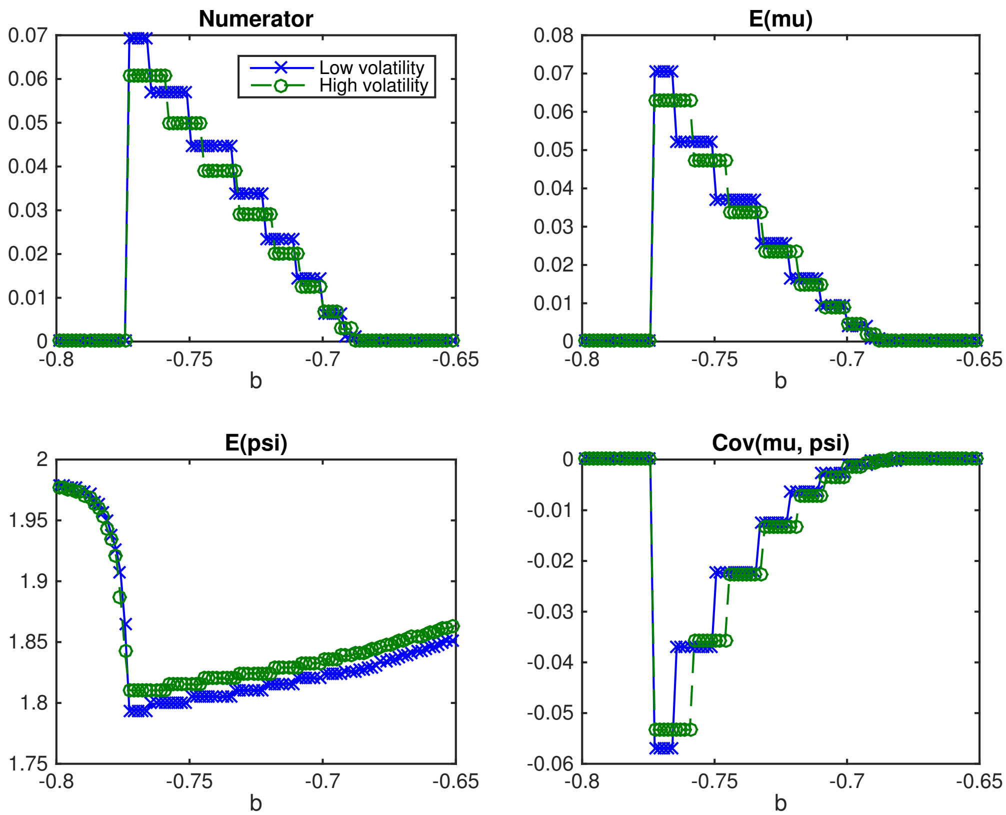

Figure 16. Decomposition of the tax on debt as a function of savings: different variances of the interest rates

interest rate takes on high levels. However, over most of the points in the savings grid, the first two effects dominate the planner's decision, so there is a higher intervention when interest rates are low.

Finally, in Figure 16, we present the decomposition for two different levels of interest rate volatility. The effects are less clear here: we can only observe a slightly higher expectation of the externality ψ when the high volatility regime prevails. This could be associated to a possible concavity of ψ with respect to interest rates, that would lead to a Jensen-inequality type of effect (i.e., if the variance of the interest rate increases, then the expectation of a concave function of the interest rate would fall). However, the stringency of the borrowing constraint, and the covariance term show a non-monotonic response with respect to volatility, that cause the non-uniform response of the macroprudential tax with respect to this shock.

The decomposition of the optimal tax on debt shows that the relations between external shocks and the incentives on the planner's problem are complex. The dynamics of the pecuniary externality and the multiplier on the borrowing constraint are determined in general equilibrium and in response to forward-looking factors, and the ultimate policy prescriptions depend upon the different forces acting in the economy. The lesson of this exercise is that simple policy prescriptions based on partial equilibrium rationales are insufficient to internalize the effect of overborrowing on asset prices and households' borrowing capacity, and they might lead to counterproductive outcomes. A better understanding the equilibrium behavior of the factors in the planner's solution would provide further insights about the operation of the economy and on the determination of appropriate policy prescriptions.

4. Conclusion

In recent years, the international capital flows entering small open economies have become larger in volume and more volatile. The uncertainty regarding policy actions in industrialized economies, as well as other underlying institutional and financial risks, have made the timing and direction of capital flows unpredictable. Policy makers around the world have grown concerned about the potential consequences of sudden reversals over their domestic financial sectors and ultimately on the real economic activity. This has motivated the surge of a myriad of unconventional policy tools to moderate the movement and regulate the composition of transborder capital flows. The international community has recognized that the risks carried by the volatility of international flows call for a more thorough analysis of the design and implementation of macroprudential capital account policies (see IMF, 2012). This work makes a contribution in our understanding of the direction and intensity of macroprudential interventions that should be undertaken when a borrowing constrained economy faces external shocks.

In the paper, we extend the small open economy framework of Jeanne and Korinek (2010) and Bianchi and Mendoza (2011) to include shocks to the level and volatility of the interest rate faced by the economy, in the spirit of Fernández-Villaverde et al. (2011). We show that the dynamics of interest rates around episodes of sudden stop generated by the model have a similar behavior to the one observed empirically in a group of emerging markets. In the model, there is scope for a Pareto improving intervention that internalizes the effect of household borrowing in the value of domestic assets, and thus in the collateral that can be used for external borrowing. The planner's intervention dictates increasing the cost of households' borrowing when it is likely that both a collateral constraint might be binding in the near future and pecuniary externalities are high. We show that the planner tends to increase his intervention as a response to low realizations of the endowment shock to offset the negative effect on the value of collateral and the tightening of the borrowing constraint that accompanies negative output realizations. The planner, on the other hand, tends to increase his intervention as a response to low interest rates shocks to offset the increase in the size of pecuniary externalities, despite the fact that there is a lower possibility of hitting a borrowing constraint. Moreover, we show that, keeping the level of interest rates constant, the planner has a non-monotonic response to interest rate volatility shocks. The degree of his intervention depends on how the changes in external volatility affect the expectations of the shadow value of collateral, pecuniary externalities, and the covariance between these two factors.

The lessons of this exercise for policy makers facing a rise in external risks are not clear-cut. We conclude that a mere increase in the volatility of external interest rates, like the one observed in recent months as the international financial markets adjust to expected policy changes in industrialized economies, does not necessarily call for a higher macroprudential intervention and the imposition of more stringent controls on the capital account. Policy makers should not only weigh the possibility of current account reversals to shape their interventions; they should also consider how external shocks affect the size of pecuniary externalities and the borrowing capacity of the country.

References

Ahmed, S. and Zlate, A. (2013). Capital flows to emerging market economies: a brave new world? International Finance Discussion Papers 1081, Board of Governors of the Federal Reserve System (U.S.).

Aizenman, J., Binici, M., and Hutchison, M. M. (2014). The transmission of federal reserve tapering news to emerging financial markets. NBER Working Papers 19980, National Bureau of Economic Research, Inc.

Aruoba, S. B., Fernandez-Villaverde, J., and Rubio-Ramirez, J. F. (2006). Comparing solution methods for dynamic equilibrium economies. *Journal of Economic dynamics and Control*, 30(12):2477–2508.

Benigno, G., Chen, H., Otrok, C., Rebucci, A., and Young, E. (2013a). Capital controls or real exchange rate policy? a pecuniary externality perspective. Research Department Publications IDB-WP-393, Inter-American Development Bank, Research Department.

Benigno, G., Chen, H., Otrok, C., Rebucci, A., and Young, E. R. (2011). Revisiting overborrowing and its policy implications. In CÈspedes, L. F., Chang, R., and Saravia, D., editors, *Monetary Policy under Financial Turbulence*, volume 16 of *Central Banking, Analysis, and Economic Policies Book Series*, chapter 6, pages 145–184. Central Bank of Chile.

Benigno, G., Chen, H., Otrok, C., Rebucci, A., and Young, E. R. (2013b). Financial crises and macro-prudential policies. *Journal of International Economics*, 89(2):453–470.

Bernanke, B. S., Gertler, M., and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. and Woodford, M., editors, *Handbook of Macroeconomics*, volume 1 of *Handbook of Macroeconomics*, chapter 21, pages 1341–1393. Elsevier.

Bianchi, J. (2011). Overborrowing and systemic externalities in the business cycle. *American Economic Review*, 101(7):3400–3426.

Bianchi, J. and Mendoza, E. G. (2011). Overborrowing, financial crises and 'macro-prudential' policy. 2011 Meeting Papers 175, Society for Economic Dynamics.

Bianchi, J. and Mendoza, E. G. (2013). Optimal time-consistent macroprudential policy. NBER Working Papers 19704, National Bureau of Economic Research, Inc.

Carrière-Swallow, Y. and Céspedes, L. F. (2013). The impact of uncertainty shocks in emerging economies. *Journal of International Economics*, 90(2):316–325.

Carroll, C. D. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics letters*, 91(3):312–320.

Chang, R. and Fernández, A. (2013). On the sources of aggregate fluctuations in emerging economies. *International Economic Review*, 54:1265–1293.

Dell'Ariccia, G., Igan, D., Laeven, L., Tong, H., Bakker, B. B., and Vandenbussche, J. (2012). Policies for macrofinancial stability: How to deal with credit booms. IMF Staff Discussion Notes 12/06, International Monetary Fund.

Eichengreen, B. and Gupta, P. (2014). Tapering talk : the impact of expectations of reduced federal reserve security purchases on emerging markets. Policy Research Working Paper Series 6754, The World Bank.

Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramírez, J. F., and Uribe, M. (2011). Risk matters: The real effects of volatility shocks. *American Economic Review*, 101(6):2530–61.

IMF (2012). The liberalization and management of capital flows: An institutional view. Technical report, International Monetary Fund.

IMF (2013). World economic outlook: Transitions and tensions. Technical report, International Monetary Fund.

Jeanne, O. and Korinek, A. (2010). Managing credit booms and busts: A pigouvian taxation approach. NBER Working Papers 16377, National Bureau of Economic Research, Inc.

Jeanne, O. and Korinek, A. (2013). Macroprudential regulation versus mopping up after the crash. NBER Working Papers 18675, National Bureau of Economic Research, Inc.

Kiyotaki, N. and Moore, J. (1997). Credit cycles. *Journal of Political Economy*, 105(2):211–48.

Klein, P., Quadrini, V., and Rios-Rull, J.-V. (2005). Optimal time-consistent taxation with international mobility of capital. *Advances in Macroeconomics*, 5(1):2.

Korinek, A. (2011). The new economics of prudential capital controls: A research agenda. *IMF Economic Review*, 59(3):523–561.

Korinek, A. and Mendoza, E. G. (2013). From sudden stops to fisherian deflation: Quantitative theory and policy implications. NBER Working Papers 19362, National Bureau of Economic Research, Inc.

Lane, P. R. and Milesi-Ferretti, G. M. (2007). The external wealth of nations mark ii: Revised and extended estimates of foreign assets and liabilities, 1970–2004. *Journal of international Economics*, 73(2):223–250.

Mackowiak, B. (2007). External shocks, u.s. monetary policy and macroeconomic fluctuations in emerging markets. *Journal of Monetary Economics*, 54(8):2512–2520.

Mendoza, E. G. (2002). Credit, prices, and crashes: Business cycles with a sudden stop. In *Preventing Currency Crises in Emerging Markets*, NBER Chapters, pages 335–392. National Bureau of Economic Research, Inc.

Mendoza, E. G. (2010). Sudden stops, financial crises, and leverage. *American Economic Review*, 100(5):1941–66.

Mendoza, E. G. and Smith, K. A. (2002). Margin calls, trading costs, and asset prices in emerging markets: The finanical mechanics of the 'sudden stop' phenomenon. NBER Working Papers 9286, National Bureau of Economic Research, Inc.

Mendoza, E. G. and Smith, K. A. (2006). Quantitative implications of a debt-deflation theory of sudden stops and asset prices. *Journal of International Economics*, 70(1):82–114.

Neumeyer, P. A. and Perri, F. (2005). Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics*, 52(2):345–380.

Ostry, J. D., Ghosh, A. R., Habermeier, K. F., Chamon, M., Laeven, L., Qureshi, M. S., and Kokenyne, A. (2011). Managing capital inflows: What tools to use? IMF Staff Discussion Notes 11/06, International Monetary Fund.

Reyes-Heroles, R. and Tenorio, G. (2015). External volatility and sudden stops in emerging markets. Technical report.

Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics letters*, 20(2):177–181.

Uribe, M. and Yue, V. Z. (2006). Country spreads and emerging countries: Who drives whom? *Journal of International Economics*, 69(1):6–36.

Appendix A. Appendix. Microeconomic foundations of the model

A.1. **The timing of borrowing and asset trading.** We denote the individual and aggregate household choice variables with lowercase and uppercase letters, respectively. We divide any given period in three sub periods: morning, afternoon, and night.

The period begins in the morning, with aggregate asset holdings (B, S) carried from the previous period. The realization of the external shocks $X = (z, r, \sigma^r)$ takes place at the beginning of the morning, and individual households receive the dividends from their holdings of the tree, $s \cdot d \exp(z)$. Each household makes an optimal consumption and portfolio decision (b', s', c) subject to its budget and borrowing constraints (2) and (3), taking the morning price $\mathcal{Q}(B, X)$ and interest rate $R \exp(r)$ as given. In this sub period, there is perfect enforcement of debt contracts, so the household fully repays its outstanding debt b before consuming. At this point, the choice of c is just a plan; every household carries the physical goods it has designated to consume into the following subperiods. In addition, we assume that the household undertakes borrowing b' with just one foreign lender. This can be justified by introducing an infinitesimally small fixed cost of borrowing with each additional competitive lender.

In the afternoon, an individual household is holding a portfolio of assets (b', s'), and has c units of consumption good to eat later at night. At this point, the household has the possibility of diverting the stocks that it holds by selling them to the rest of the households in the economy and defaulting on his outstanding debt with the foreign lender. The defaulting household, however, cannot steal the entirety of the asset; it can only take a fraction $(1-\kappa) \in [0, 1]$ away, and it leaves behind the remaining of its holdings. We denote by $\mathcal{Q}^c(B, X)$ the prevailing price for this transaction in the afternoon market. Since we assume that households compete à la Bertrand for the stocks of the tree, then the market price of the tree in the afternoon is as high as the representative household prices the dividend payouts and resale value next period, according to the following Euler equation:

$$\mathcal{Q}^c(B, X) = \beta \mathbb{E} \left[\left. \frac{u'(\mathcal{C}(\mathcal{B}(B, X), X'))[\mathcal{Q}(\mathcal{B}(B, X), X') + d(X')]}{u'(\mathcal{C}(B, X))} \right| X \right],$$

where $\mathcal{C}$ and $\mathcal{B}$ denote the aggregate decision rules of the economy.

At night, the international lender finds out whether he has been defaulted or not. If he has, he is entitled to obtain the fraction κ of the household's stockholdings that were not diverted. The lender, nevertheless, cannot directly receive dividends from the tree, so he must necessarily sell it to the local households in order to obtain a profit. Again, households compete à la Bertrand to buy the banker's tree holdings, so the value at which the transaction takes place is the prevailing market price, $\mathcal{Q}^c(B, X)$. The lender then proceeds to loan the receipts of the transaction in the international financial market, at the prevailing risk-free interest rate, $R \exp(r)$. Since the interest rate is positive, and the evolution of the stock prices does not in general have a positive trend, the lender has incentives to immediately sell the stocks and lend the revenue in the overnight market.[13] After these transactions take place, the non-defaulting households are able to consume what they had originally planned, c.

In order to avoid losses from household default, lenders constrain the amount that they lend, $-b'/R\exp(r)$, to be less than or equal to the market value of the household's asset holdings that cannot be diverted, $\kappa\mathcal{Q}^c(B,X)s'$. This justifies the presence of the borrowing constraint (3) in the problem of the representative household.

It only remains to explain the relation between the morning and the afternoon prices, $\mathcal{Q}$ and $\mathcal{Q}^c$. Suppose that the borrowing constraint is binding, so $\mu(B,X)>0$. For every additional stock of the tree that the household buys in the morning, it must sacrifice $\mathcal{Q}(B,X)$ units of consumption, that are valued at the marginal utility $u'(B,X)$. On the other hand, by buying more stocks of the tree, the representative household relaxes the borrowing constraint, and obtains a marginal benefit of $\kappa\mu(B,X)\mathcal{Q}^c(B,X)$, in the same sub period. Thus, the net marginal cost of saving in stocks of the tree in the morning is:

$$\mathcal{Q}(B,X)u'(\mathcal{C}(B,X))-\kappa\mu(B,X)\mathcal{Q}^c(B,X).$$

In the afternoon, the household can sell these stocks at the prevailing price, $\mathcal{Q}^c(B,X)$, which is valued at the marginal utility of consumption $u'(\mathcal{C}(B,X))$. Thus, for the household demand of stocks to be optimal, it must be the case that the marginal cost in the morning equates the marginal benefit in the afternoon:

$$\mathcal{Q}(B,X)u'(\mathcal{C}(B,X))-\kappa\mu(B,X)\mathcal{Q}^c(B,X)=\mathcal{Q}^c(B,X)u'(\mathcal{C}(B,X)).$$

From this expression, it is easy to see that whenever the borrowing constraint binds, the value of the tree in the morning will be higher than in the afternoon, because it helps the households relax the borrowing constraint and increase their debt. The decrease in prices from the morning to the afternoon is perfectly foreseen by every agent in the economy, but there are no opportunities of arbitrage because it is forbidden to hold the asset in short positions.

A.1.1. *The planner's intervention.* The social planner understands that the current aggregate level of debt, B, and the choice of future indebtedness B' affect the value of collateral available in the economy, and thus constrain the borrowing possibilities of the households. In order to internalize this pecuniary externality, the planner can control the households' borrowing decisions that take place in the morning, B.

Nonetheless, the planner cannot overcome the fact that households can divert their asset holdings in the afternoon and default on their outstanding debt. Moreover, the planner cannot intervene in the night stock market, in which the defaulted foreign lenders sell the remaining fractions of the diverted asset. Thus, the planner faces the same borrowing constraint as the households (3), and the price of the assets must be consistent with the household Euler

[13] Otherwise, we can assume that the holdings of the tree depreciate overnight when held by the lender, so he has incentives to immediately sell them.

equation of stocks:

$$\mathcal{Q}(B,X)=\beta\mathbb{E}\left[\frac{u'(\mathcal{C}(\mathcal{B}(B,X),X'))[\mathcal{Q}(\mathcal{B}(B,X),X')+d(X')]}{u'(\mathcal{C}(B,X))}\middle| X\right].$$

In this case, the market price of the stocks is the same throughout the day, because households do not internalize the effect of their savings in stocks on the borrowing possibilities for the planner's problem.

A.2. **Competitive equilibrium.** Consider the recursive formulation of the household's problem, expressed in program (4). The solution to the household's problem is characterized by a pair of optimal decision rules for bonds and stocks, $\hat{b}\left(b,s,B,X\right)$ and $\hat{s}\left(b,s,B,X\right)$ respectively, that satisfy the following set of equations:

$$\begin{aligned} u'\left(c\right) &= \mu\left(b,s,B,X\right)+\beta R\left(X\right)\mathbb{E}\left[u'\left(c'\right)|X\right], \text{ and} \\ \mathcal{Q}\left(B,X\right)u'\left(c\right) &= \beta\mathbb{E}\left[u'\left(c'\right)\left(\mathcal{Q}\left(\mathcal{B}\left(B,X\right),X'\right)+d\left(X'\right)\right)|X\right] \\ &+ \mathcal{Q}^{c}\left(B,X\right)\mu\left(b,s,B,X\right)\kappa, \end{aligned}$$

the budget constraint of the household in each period, and the collateral constraint

$$-\frac{\hat{b}\left(b,s,B,X\right)}{R\left(X\right)}\leq\kappa\mathcal{Q}^{c}\left(B,X\right)\hat{s}\left(b,s,B,X\right).$$

We now proceed to define a recursive competitive equilibrium.

Definition 1. *A recursive competitive equilibrium of this economy consists of pricing functions $\hat{\mathcal{Q}}\left(B,X\right)$ and $\hat{\mathcal{Q}}^{c}\left(B,X\right)$, a perceived law of motion for aggregate bond holdings, $\hat{\mathcal{B}}\left(B,X\right)$, and decision rules for households, $\hat{b}\left(b,s,B,X\right)$ and $\hat{s}\left(b,s,B,X\right)$, with associated value function $\hat{V}\left(b,s,B,X\right)$ such that:*

1. *Given $\hat{\mathcal{Q}}\left(B,X\right)$, $\hat{\mathcal{Q}}^{c}\left(B,X\right)$ and $\mathcal{B}\left(B,X\right)$, households' decision rules, $\hat{b}\left(b,s,B,X\right)$ and $\hat{s}\left(b,s,B,X\right)$, and the associated value function $\hat{V}\left(b,s,B,X\right)$ solve the recursive problem of the household given by (4).*
2. *$\hat{\mathcal{B}}\left(B,X\right)$ is consistent with the actual law of motion for bond holdings; $\hat{\mathcal{B}}\left(B,X\right)=\hat{b}\left(B,1,B,X\right)$.*
3. *Markets must clear. In particular, $\hat{\mathcal{Q}}\left(B,X\right)$ and $\hat{\mathcal{Q}}^{c}\left(B,X\right)$ are such that $\hat{s}(B,1,B,X)=1$.*

Given the definition of the equilibrium, notice that the equilibrium level of bonds can be characterized by a simple function of the aggregate state variables, $B'=\hat{\mathcal{B}}\left(B,X\right)$, which together with the resource constraint defines consumption as a function of aggregate state variables, $\hat{\mathcal{C}}\left(B,X\right)$.

A.3. **Social planner's recursive problem.** We consider a social planner that lacks commitment and that can only choose aggregate bond holdings for households, but is still subject to the borrowing constraint. Following Klein et al. (2005), in order to solve for the time consistent policy, we focus on Markov stationary policy rules that only depend on the current state of the economy. In particular, they only depend on the aggregate state of the economy, (B, X). We solve for the constrained efficient allocation following the three steps described in Klein et al. (2005): (i) we first define a recursive competitive equilibrium for arbitrary policy rules; (ii) we then proceed to define a constrained-efficient allocation for arbitrary policy rules of future planners; and (iii) we define the constrained efficient allocation for the case in which such policies are time consistent, i.e., we solve for the fixed point of the game being played by successive planners. In this problem, the social planner makes the borrowing decisions for the households, so he is the one facing the collateral constraint. Households are allowed to trade stocks of the tree freely, without government intervention.

Let us consider a planner who chooses an arbitrary sequence of state-contingent lump-sum transfers, $\{T_t\}_{t=0}^{\infty}$. Given this sequence of transfers, we can write down the Bellman equation for the household's problem as follows:

$$V^A(s, T, X) = \max_{c, s'} \left\{ u(c) + \beta \mathbb{E}\left[V^A\left(s', T', X'\right) | X \right] \right\}$$

subject to

$$c + \mathcal{Q}^A(T, X)\, s' = \left[\mathcal{Q}^A(T, X) + d(X) \right] s + T.$$

When solving this problem, the household takes the pricing function, $\mathcal{Q}^A(T, X)$, and the sequence of transfers as given. The solution to this problem is characterized by a policy rule for stock holdings, $s^A(s, T, X)$, such that Euler equation for stock holdings holds,

$$\mathcal{Q}^A(T, X) = \frac{\beta \mathbb{E}\left[u'(c') \left(\mathcal{Q}^A(T', X') + d(X') \right) | X \right]}{u'(c)},$$

where

$$c + \mathcal{Q}^A(T, X)\, s^A(s, T, X) = \left[\mathcal{Q}^A(T, X) + d(X) \right] s + T.$$

Notice that the resource constraint of the economy implies that $T = B - \frac{B'}{R(X)}$. Hence, given B, the planner actually chooses T by choosing B'. Therefore, we can rewrite the planner's policy rule as one that dictates B' as a function of the current aggregate state, (B, X). Call this policy rule $\Psi(B, X)$, and define the following functions:

$$\begin{aligned}
\mathcal{Q}(B, X) &\equiv \mathcal{Q}^A\left(B - \frac{\Psi(B, X)}{R(X)}, X \right), \\
s(s, B, X) &\equiv s^A\left(s, B - \frac{\Psi(B, X)}{R(X)}, X \right), \text{ and} \\
V(s, B, X) &\equiv V^A\left(s, B - \frac{\Psi(B, X)}{R(X)}, X \right).
\end{aligned}$$

Hence, we can rewrite the optimality conditions for the household's problem as follows:

$$\mathcal{Q}(B, X) = \frac{\beta \mathbb{E}\left[u'(c') \left(\mathcal{Q}(B', X') + d(X') \right) | X \right]}{u'(c)},$$

where

$$c+\mathcal{Q}(B,X)\,\hat{s}(s,T,X)=\left[\mathcal{Q}(B,X)+d(X)\right]s+B-\frac{\Psi(B,X)}{R(X)}.$$

Definition 2. *A recursive competitive equilibrium for an arbitrary policy rule $\Psi(B,X)$ consists of a pricing function, $\hat{\mathcal{Q}}(B,X)$, and decision rules for households, $\hat{s}(s,B,X)$, with associated value function $\hat{V}(s,B,X)$ such that:*

1. *Given $\Psi(B,X)$ and $\hat{\mathcal{Q}}(B,X)$, households' decision rules, $\hat{s}(s,B,X)$, and the associated value function $\hat{V}(s,B,X)$ solve the recursive problem of the household.*
2. *Markets clear: $\hat{\mathcal{Q}}(B,X)$ is such that $\hat{s}(s,B,X)=1$ and the resource constraint holds, $c+\frac{B'}{R(X)}=B+d(X)$, where $B'=\Psi(B,X)$.*

Therefore, in such an equilibrium, we have that the following set of equations must be satisfied:

$$\begin{aligned}\hat{\mathcal{Q}}(B,X) &= \frac{\beta\mathbb{E}\left[u'\left(B'+d(X)-\frac{B''}{R(X)}\right)\left[\hat{\mathcal{Q}}(B',X')+d(X')\right]\Big|\,X\right]}{u'\left(B+d(X)-\frac{B'}{R(X)}\right)},\\ B' &= \Psi(B,X) \text{ and } B''=\Psi\left(\Psi(B,X),X'\right).\end{aligned}$$

Given that the planner we consider can only affect the allocation of bond holdings, but cannot directly intervene in the markets for stocks, the pricing condition for $\hat{\mathcal{Q}}(B,X)$ has to hold in a constrained efficient allocation, in particular, this condition defines the price at which lenders value collateral in the current period borrowing constraint. Taking into account this kind of implementability constraint for the planner, we can now define the problem to be solved by a planner that takes as given the policy functions of future planners. Given future policy rules, $\Psi(B,X)$, associated pricing function $\hat{\mathcal{Q}}(B,X)$, and consumption rule $\mathcal{C}(B,X)$, the current planner chooses current consumption, c, and future bond holdings to solve the following Bellman equation:

$$W(B,X)=\max_{c,B'}\left\{u(c)+\beta\mathbb{E}\left[W\left(B',X'\right)|X\right]\right\}$$

subject to

$$\begin{aligned}c+\frac{B'}{R(X)} &= d(X)+B,\\ -\frac{B'}{R(X)} &\leq \kappa\tilde{\mathcal{Q}}(c,B',X),\end{aligned}$$

where

$$\tilde{\mathcal{Q}}(c,B',X)=\frac{\beta\mathbb{E}\left[u'\left(\mathcal{C}(B',X')\right)\left(\mathcal{Q}(B',X')+d(X')\right)|X\right]}{u'(c)},$$

and $\mathcal{C}(B',X')=d(X')+B'-\frac{\Psi(B',X')}{R(X')}$.

Definition 3. *A constrained efficient allocation given a policy rule for future planners* $\Psi(B,X)$*, with associated pricing function* $\hat{\mathcal{Q}}(B,X)$ *and consumption rule* $\mathcal{C}(B,X)$*, consists of an optimal policy rule,* $\hat{\Psi}(B,X)$*, such that given functions* $\Psi(B,X)$*,* $\hat{\mathcal{Q}}(B,X)$ *and* $\mathcal{C}(B,X)$*, the current policy rule* $B' = \hat{\Psi}(B,X)$ *and associated value function,* $\hat{W}(B,X)$*, solve the recursive problem of the current planner.*

Let us define the following function,

$$\bar{\mathcal{Q}}\left(B,B',X\right) = \beta\mathbb{E}\left[\left.\frac{u'\left(B' + d\left(X'\right) - \frac{\Psi(B',X')}{R(X')}\right)\left[\hat{\mathcal{Q}}\left(B',X'\right) + d\left(X'\right)\right]}{u'\left(d\left(X\right) + B - \frac{B'}{R(X)}\right)}\right| X\right].$$

Then, $\hat{\Psi}(B,X)$ has to be such that the generalized Euler equation holds:

$$\begin{aligned} u'(\hat{\mathcal{C}}(B,X)) - \hat{\mu}(B,X)\left[1 + \kappa R(X)\,\xi(B,X)\right] \\ = R(X)\,\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\hat{\mu}(B',X')\psi(B',X')|X\right], \end{aligned} \tag{10}$$

where $\psi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B}$, $\xi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B'}$, and $\hat{\mathcal{C}}(B,X) = B + d(X) - \frac{\hat{\Psi}(B,X)}{R(X)}$. The multiplier on the collateral constraint is given by:

$$\begin{aligned} \hat{\mu}(B,X) = \max\Bigg\{0, \frac{1}{1+\kappa R(X)\xi(B,X)}\Bigg[u'\left(B + d(X) - \frac{\hat{\Psi}(B,X)}{R(X)}\right) \\ - \beta R(X)\,\mathbb{E}\left[u'\left(\mathcal{C}\left(B',X'\right)\right) + \kappa\mu\left(B',X'\right)\psi\left(B',X'\right)|X\right]\Bigg]\Bigg\}, \end{aligned}$$

where $\hat{\Psi}(B,X) = -R(X)\,\kappa\bar{\mathcal{Q}}(B,\Psi(B,X),X)$. After this characterization of the allocation, we can now define a recursive constrained efficient allocation as follows.

Definition 4. *The recursive constrained efficient allocation consists of functions* $\Psi(B,X)$*,* $\hat{\mathcal{Q}}(B,X)$*,* $\mathcal{C}(B,X)$*, and* $\hat{\Psi}(B,X)$ *with associated value function,* $\hat{W}(B,X)$*, such that:*

1. $\hat{\mathcal{Q}}(B,X)$*,* $\mathcal{C}(B,X)$*,* $\hat{\Psi}(B,X)$*, and the associated value function* $\hat{W}(B,X)$*, constitute a constrained efficient allocation, given a policy rule for future planners,* $\Psi(B,X)$*.*
2. *The planner's plans are time-consistent:* $\hat{\Psi}(B,X) = \Psi(B,X)$ *and* $\bar{\mathcal{Q}}(B,\hat{\Psi}(B,X),X) = \hat{\mathcal{Q}}(B,X)$*.*

A.3.1. *Non-binding current collateral constraint:* $\mu(B,X) = 0$*.* Let us consider first the case in which $\mu(B,X) = 0$. Given our definition of $\bar{\mathcal{Q}}(B,B',X)$, notice that:

$$\begin{aligned} \frac{\partial\bar{\mathcal{Q}}\left(B,B',X\right)}{\partial B} &= \beta\mathbb{E}\left\{-\frac{u'\left(\mathcal{C}\left(B',X'\right)\right)\left(\hat{\mathcal{Q}}\left(B',X'\right) + d\left(X'\right)\right)}{u'\left(c\right)}\frac{u''\left(c\right)}{u'\left(c\right)}\right\} \\ &= -\frac{u''\left(c\right)}{u'\left(c\right)}\bar{\mathcal{Q}}\left(B,B',X\right), \end{aligned}$$

which implies that:

$$\psi\left(B,X\right)=-\frac{u''\left(\mathcal{C}\left(B,X\right)\right)}{u'\left(\mathcal{C}\left(B,X\right)\right)}\hat{\mathcal{Q}}\left(B,X\right).$$

Therefore, when $\mu\left(B,X\right)=0$, condition (10) becomes a regular Euler equation (with a μ wedge):

$$u'\left(\hat{\mathcal{C}}\left(B,X\right)\right)=R\left(X\right)\beta\mathbb{E}\left[u'\left(\mathcal{C}\left(B',X'\right)\right)-\kappa\hat{\mu}\left(B',X'\right)\frac{u''\left(\mathcal{C}\left(B',X'\right)\right)}{u'\left(\mathcal{C}\left(B',X'\right)\right)}\hat{\mathcal{Q}}\left(B',X'\right)|X\right].$$

A.3.2. *Binding current collateral constraint:* $\mu\left(B,X\right)>0$. Let us first notice that the current planner has to choose B' subject to the collateral constraint

$$\frac{B'}{R\left(X\right)}+\kappa\bar{\mathcal{Q}}\left(B,B',X\right)\geq 0.$$

If the left hand side of the previous inequality is strictly increasing in B', then, given B, there is a unique B' such that this equation holds with equality. Hence, when the current collateral constraint is binding, the optimal policy rule by the current planner must solve $\frac{\hat{\Psi}(B,X)}{R(X)}+\kappa\bar{\mathcal{Q}}(B,\hat{\Psi}(B,X),X)=0$, and this policy rule is unique. Notice that that left hand side if strictly increasing if and only if

$$1+\kappa R\left(X\right)\xi\left(B,X\right)>0.$$

In equilibrium, $\xi\left(B,X\right)<0$, therefore we expect this condition to hold whenever κ is a small number.[14] Given the definition of $\bar{Q}\left(B,B',X\right)$, notice that

$$\frac{\partial\bar{\mathcal{Q}}\left(B,B',X\right)}{\partial B'}=\frac{\beta\mathbb{E}\left[\Omega\left(B,B',X\right)\right]}{u'\left(c\right)}+\frac{u''\left(c\right)}{u'\left(c\right)}\frac{\bar{\mathcal{Q}}\left(B,B',X\right)}{R\left(X\right)}\tag{11}$$

where

$$\Omega(B,B',X)=u''(\mathcal{C}(B',X'))\frac{\partial\mathcal{C}(B',X')}{\partial B}[\mathcal{Q}(B',X')+d(X')]+u'(\mathcal{C}(B',X'))\frac{\partial\mathcal{Q}(B',X')}{\partial B}.$$

This last expression shows how the current planner takes into account how his decision affect future planners actions by changing B'.

Appendix B. Appendix. Numerical solution of the model

B.1. **Competitive equilibrium.** Let us denote by B the aggregate equilibrium savings of the economy, and by $X=(z,r,\sigma^r)$ the realization of exogenous shocks. We wish to find

[14]Notice that when κ is small enough, then the term $\kappa\mu\left(B',X'\right)\psi\left(B',X'\right)$ also becomes very small and $\hat{\Psi}\left(B,X\right)$ is also uniqu in the case in which $\mu\left(B,X\right)=0$.

functions $\mathcal{B}(B,X)$, $\mathcal{C}(B,X)$, $\mathcal{Q}(B,X)$, $\mathcal{Q}^c(B,X)$ and $\mu(B,X)$ that satisfy:

$$u'(\mathcal{C}(B,X)) = \beta R(X)\mathbb{E}\left[u'(\mathcal{C}(\mathcal{B}(B,X),X'))|X\right] + \mu(B,X), \tag{12}$$

$$\mathcal{C}(B,X) + \frac{\mathcal{B}(B,X)}{R(X)} = d(X) + B, \tag{13}$$

$$-\frac{\mathcal{B}(B,X)}{R(X)} \leq \kappa\mathcal{Q}(B,X), \tag{14}$$

$$\mathcal{Q}^c(B,X) = \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}(\mathcal{B}(B,X),X'))\left[\mathcal{Q}(\mathcal{B}(B,X),X') + d(X')\right]}{u'(\mathcal{C}(B,X)) - \kappa\mu(B,X)}\right| X\right], \tag{15}$$

$$\mathcal{Q}(B,X) = \left(1 + \frac{\kappa\mu(B,X)}{u'(\mathcal{C}(B,X))}\right)\mathcal{Q}^c(B,X). \tag{16}$$

We extend the endogenous grid method (EGM) of Carroll (2006) to our framework where there is a borrowing constraint that binds occasionally:

1. For each $\sigma^r \in \{\sigma_L^r, \sigma_H^r\} \equiv \mathcal{S}$, calculate the transition matrix for a discrete approximation to the VAR(1) process of (z,r) over $\mathcal{Z}\times\mathcal{R}$, with $\mathcal{Z} = \{z_1,\ldots,z_{Nz}\}$ and $\mathcal{R} = \{r_1,\ldots,r_{Nz}\}$.
2. Generate a grid $\bar{\mathcal{B}} = \{b_1, b_2, \ldots, b_N\}$, and an extended grid
$$\bar{\bar{\mathcal{B}}} = \bar{\mathcal{B}} \cup \{b_{N+1}, b_{N+2}, \ldots, b_{N+M}\},$$
where b_{N+M} is chosen such that the resulting $\max_X \mathcal{B}(b_N, X) \leq b_{N+M}$ (to be verified in the end).
3. Guess functions $\mathcal{C}_1(B,X)$, $\mathcal{Q}_1(B,X)$ and $\mathcal{Q}_1^c(B,X)$, for every $(B,X) \in \bar{\bar{\mathcal{B}}}\times\mathcal{Z}\times\mathcal{R}\times\mathcal{S}$. The initial guess we use is:
$$\mathcal{C}_1(B,X) = d(X) + B\left(1 - \frac{1}{R(X)}\right),$$
$$\mathcal{Q}_1(B,X) = \frac{\beta}{1-\beta}d(X),$$
and $\mathcal{Q}_1^c(B,X) = \mathcal{Q}_1(B,X)$, which corresponds to the assumption that $\mathcal{B}(B,X) = B$, $z' = z$ and $r' = r$ for all (B,X).
4. Set $\mathcal{C}_0(B,X) = \mathcal{C}_1(B,X)$, $\mathcal{Q}_0(B,X) = \mathcal{Q}_1(B,X)$ and $\mathcal{Q}_0^c(B,X) = \mathcal{Q}_1^c(B,X)$ for each $(B,X) \in \bar{\bar{\mathcal{B}}}\times\mathcal{Z}\times\mathcal{R}\times\mathcal{S}$.
5. Assume that (14) does not bind. Use (12) and (13) to calculate:
$$\hat{\mathcal{C}}(B',X) = {u'}^{-1}\left(\beta R(X)\mathbb{E}\left[u'(\mathcal{C}_0(B',X'))|X\right]\right),$$
$$\hat{\mathcal{B}}(B',X) = \hat{\mathcal{C}}(B',X) + \frac{B'}{R(X)} - d(X).$$
Notice that $\hat{\mathcal{B}}$ is the level of contemporaneous savings that yield an optimal savings decision B' when the realization of shocks is X and the borrowing constraint does not bind.

6. For each X, let us denote by $\bar{\hat{\mathcal{B}}}(X)$ the endogenous grid of points generated by $\hat{\mathcal{B}}(B', X)$. For every X, interpolate B' from $\hat{\mathcal{B}}(B', X)$ to $\bar{\mathcal{B}}$, and denote the resulting function $\check{B}(B, X)$.
7. Calculate $\tilde{\mathcal{B}}(B, X) = \max\{\check{B}(B, X), -\kappa R(X)\mathcal{Q}_0^c(B, X)\}$, and the corresponding consumption:
$$\tilde{\mathcal{C}}(B, X) = d(X) + B - \frac{\tilde{\mathcal{B}}'(B, X)}{R(X)}.$$
8. Find $\mathcal{B}^*(B, X) = \min\{B \in \bar{\bar{\mathcal{B}}} : B \geq \tilde{\mathcal{B}}(B, X)\}$. Using (12), (15) and (16), find:
$$\tilde{\mu}(B, X) = u'(\tilde{\mathcal{C}}(B, X)) - \beta R(X)\mathbb{E}\left[u'(\mathcal{C}_0(\mathcal{B}^*(B, X), X'))|X\right],$$
$$\tilde{\mathcal{Q}}^c(B, X) = \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}_0(\mathcal{B}^*(B, X), X'))\left[\mathcal{Q}_0(\mathcal{B}^*(B, X), X') + d(X')\right]}{u'(\tilde{\mathcal{C}}(B, X)) - \kappa\tilde{\mu}(B, X)}\right| X\right],$$
$$\tilde{\mathcal{Q}}(B, X) = \left(1 + \frac{\kappa\tilde{\mu}(B, X)}{u'(\tilde{\mathcal{C}}(B, X))}\right)\tilde{\mathcal{Q}}^c(B, X).$$
9. For every $(B, X) \in \bar{\mathcal{B}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$, update:
$$\mathcal{C}_1(B, X) = \alpha\tilde{\mathcal{C}}(B, X) + (1 - \alpha)\mathcal{C}_0(B, X),$$
$$\mathcal{Q}_1(B, X) = \alpha\tilde{\mathcal{Q}}(B, X) + (1 - \alpha)\mathcal{Q}_0(B, X)$$
$$\mathcal{Q}_1^c(B, X) = \alpha\tilde{\mathcal{Q}}^c(B, X) + (1 - \alpha)\mathcal{Q}_0^c(B, X).$$
for some $\alpha \in (0, 1]$. For $B \in \bar{\bar{\mathcal{B}}}\backslash\bar{\mathcal{B}}$, set $\mathcal{C}_1(B, X) = \mathcal{C}_1(b_N, X)$, $\mathcal{Q}_1(B, X) = \mathcal{Q}_1(b_N, X)$ and $\mathcal{Q}_1^c(B, X) = \mathcal{Q}_1^c(b_N, X)$.
10. Repeat steps 4-9 until convergence.

B.2. **Constrained efficient allocation.** The constrained efficient allocation satisfies:

$$u'(\mathcal{C}(B, X)) - \mu(B, X)\left[1 + \kappa R(X)\xi(B, X)\right] = R(X)\beta\mathbb{E}\left[u'(\mathcal{C}(B', X')) + \kappa\mu(B', X')\psi(B', X')|X\right], \tag{17}$$
$$\mathcal{Q}(B, X) = \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}(\mathcal{B}(B, X), X'))\left[\mathcal{Q}(\mathcal{B}(B, X), X') + d(X')\right]}{u'(\mathcal{C}(B, X)) - \kappa\mu(B, X)}\right| X\right], \tag{18}$$

together with (13) and (14). Some steps of the EGM algorithm change with respect to the solution of the competitive equilibrium:

3. Guess functions $\mathcal{C}_1(B, X)$, $\mathcal{Q}_1(B, X)$ and $\mu_1(B, X)$ for every $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$. The initial guess we use is: $\mu_1(B, X) = 0$.
4. Set $\mathcal{C}_0(B, X) = \mathcal{C}_1(B, X)$, $\mathcal{Q}_0(B, X) = \mathcal{Q}_1(B, X)$ and $\mu_0(B, X) = \mu_1(B, X)$ for each $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$.

   Calculate:
$$\psi(B, X) = -\frac{u''(\mathcal{C}_0(B, X))}{u'(\mathcal{C}_0(B, X))}\mathcal{Q}_0(B, X).$$
   Use the numerical derivatives of C_0 and Q_0 with respect to B to calculate $\xi(B, X)$ using equation (11) of Appendix A.3.

5. Assume that (14) does not bind. Use (17) and (13) to calculate:

$$\hat{\mathcal{C}}(B',X) = u'^{-1}\left(\beta R(X)\mathbb{E}\left[u'(\mathcal{C}_0(B',X')) + \kappa\mu_0(B',X')\psi(B',X')|X\right]\right),$$
$$\hat{\mathcal{B}}(B',X) = \hat{\mathcal{C}}(B',X) + \frac{B'}{R(X)} - d(X).$$

8. Find $\mathcal{B}^*(B,X) = \min\{B \in \bar{\bar{\mathcal{B}}} : B \geq \tilde{\mathcal{B}}(B,X)\}$. Using (17) and (18), find:

$$\tilde{\mu}(B,X) = \frac{1}{1+\kappa R(X)\xi(B,X)}\Big\{u'(\tilde{\mathcal{C}}(B,X))$$
$$- \beta R(X)\mathbb{E}\left[u'(\mathcal{C}_0(\mathcal{B}^*(B,X),X')) + \kappa\mu_0(\mathcal{B}^*(B,X),X')\psi(\mathcal{B}^*(B,X),X')|X\right]\Big\},$$
$$\tilde{\mathcal{Q}}(B,X) = \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}_0(\mathcal{B}^*(B,X),X'))\left[\mathcal{Q}_0(\mathcal{B}^*(B,X),X') + d(X')\right]}{u'(\tilde{\mathcal{C}}(B,X)) - \kappa\tilde{\mu}(B,X)}\right| X\right],$$

9. For every $(B,X) \in \bar{\mathcal{B}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$, update:

$$\mathcal{C}_1(B,X) = \alpha\tilde{\mathcal{C}}(B,X) + (1-\alpha)\mathcal{C}_0(B,X),$$
$$\mathcal{Q}_1(B,X) = \alpha\tilde{\mathcal{Q}}(B,X) + (1-\alpha)\mathcal{Q}_0(B,X)$$
$$\mu_1(B,X) = \alpha\tilde{\mu}(B,X) + (1-\alpha)\mu_0(B,X).$$

for some $\alpha \in (0,1]$. For $B \in \bar{\bar{\mathcal{B}}}\backslash\bar{\mathcal{B}}$, set $\mathcal{C}_1(B,X) = \mathcal{C}_1(b_N,X)$, $\mathcal{Q}_1(B,X) = \mathcal{Q}_1(b_N,X)$ and $\mu_1(B,X) = \mu_1(b_N,X)$.

10. Repeat steps 4-9 until convergence.

B.3. **Accuracy of the approximation.** We compute the Euler equation errors following Aruoba et al. (2006) to assess the accuracy of our solution. The histograms in Figure 17 show that the errors remain below 10^{-2} units of consumption in most of the state space. The maximum levels of the errors are reached around the region where the borrowing constraint binds. The errors are modestly higher in the solution to the constrained efficient allocation, but they remain within a reasonable level.

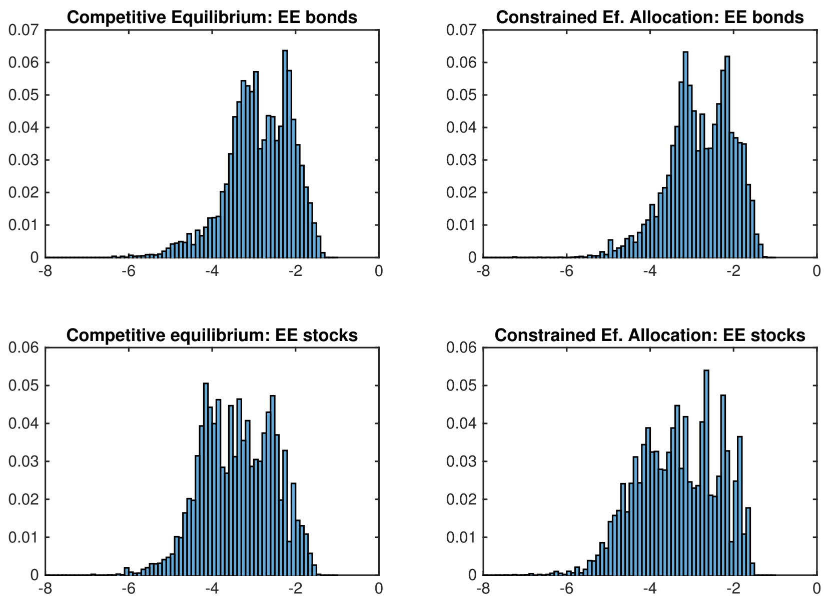

Figure 17. Euler equation errors: ergodic distributions