Online Appendix

Macroprudential Policy in the Presence of External Risks*

Ricardo Reyes-Heroles†
Federal Reserve Board

Gabriel Tenorio‡
Bank of America Securities

June, 2020

*The views in this appendix are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System or Bank of America Securities.

†Division of International Finance, Federal Reserve Board, 20th and C Streets N.W., Washington, D.C. 20551, U.S.A. E-mail: `ricardo.m.reyes-heroles@frb.gov`

‡Research Division, Bank of America Securities, One Bryant Park, New York, N.Y. 10036, U.S.A. E-mail: `gabriel.tenorio@bofa.com`

A Empirical Analysis

A.1 Country-specific Cross-correlations of Output with Risks

Table A.1: Cross-correlation of output gap with interest rate *levels*

	Correlation of output gap (t) with interest rate level ($t+j$)								
	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
Argentina	-0.18	-0.28	-0.39	-0.46	-0.49	-0.47	0.43	-0.35	-0.28
Brazil	-0.08	-0.11	-0.09	-0.07	-0.04	0.05	0.07	0.05	0.03
Bulgaria	-0.25	-0.18	-0.10	-0.03	0.09	0.12	0.15	0.17	0.13
Chile	-0.09	-0.01	0.08	0.14	0.19	0.25	0.27	0.26	0.20
Colombia	-0.25	-0.27	-0.27	-0.24	-0.18	-0.09	-0.05	-0.02	0.01
Ecuador	-0.35	-0.44	-0.47	-0.46	-0.39	-0.32	-0.25	-0.16	-0.06
Egypt	0.12	-0.02	-0.16	-0.25	-0.30	-0.32	-0.36	-0.45	-0.49
Hungary	-0.07	-0.08	-0.09	-0.10	-0.04	0.11	0.25	0.27	0.20
Indonesia	-0.23	-0.34	-0.39	-0.30	-0.12	0.08	0.25	0.31	0.29
Malaysia	-0.13	-0.19	-0.27	-0.35	-0.31	-0.17	-0.02	0.15	0.23
Mexico	-0.07	-0.14	-0.21	-0.25	-0.18	-0.04	0.05	0.12	0.17
Peru	-0.20	-0.20	-0.21	-0.18	-0.08	-0.04	0.02	0.06	0.07
Philippines	0.06	-0.01	-0.06	-0.08	-0.05	0.00	0.05	0.09	0.11
Poland	0.27	0.32	0.31	0.29	0.22	0.23	0.23	0.23	0.19
Russia	0.04	-0.05	-0.16	-0.31	-0.45	-0.47	-0.42	-0.34	-0.29
South Africa	0.12	0.07	0.05	0.07	0.13	0.19	0.23	0.24	0.23
Turkey	-0.09	-0.08	-0.02	0.07	0.14	0.25	0.32	0.28	0.24
Ukraine	-0.20	-0.25	-0.29	-0.37	-0.35	-0.13	0.08	0.22	0.33
Uruguay	-0.22	-0.33	-0.55	-0.72	-0.73	-0.61	-0.46	-0.35	-0.26
Venezuela	-0.19	-0.26	-0.29	-0.27	-0.23	-0.08	0.06	0.15	0.20
Korea	-0.16	-0.31	-0.45	-0.53	-0.50	-0.42	-0.30	-0.14	-0.03
El Salvador	0.23	0.21	0.14	0.07	0.15	0.29	0.38	0.38	0.32
Dominican Republic	-0.02	-0.19	-0.31	-0.44	-0.41	-0.28	0.03	0.30	0.40
Average	-0.08	-0.14	-0.18	-0.21	-0.17	-0.08	0.01	0.06	0.08
90th Percentile	0.12	0.05	0.07	0.07	0.15	0.24	0.26	0.29	0.32
10th Percentile	-0.25	-0.32	-0.44	-0.46	-0.48	-0.46	-0.41	-0.35	-0.28

Table A.2: Cross-correlation of output gap with interest rate *volatility*

	Correlation of output gap (t) with volatility of interest rate ($t+j$)								
	$t-4$	$t-3$	$t-2$	$t-1$	t	$t+1$	$t+2$	$t+3$	$t+4$
Argentina	-0.14	-0.18	-0.24	-0.27	-0.18	-0.15	-0.11	-0.05	-0.04
Brazil	-0.22	-0.21	-0.17	-0.07	-0.01	0.00	-0.03	0.00	0.05
Bulgaria	-0.16	-0.13	-0.17	-0.07	0.05	-0.01	-0.06	0.05	0.04
Chile	-0.42	-0.47	-0.42	-0.35	-0.25	-0.06	0.08	0.14	0.18
Colombia	-0.23	-0.33	-0.39	-0.34	-0.21	0.02	0,07	0.18	0.26
Ecuador	-0.39	-0.43	-0.42	-0.32	-0.25	-0.22	-0.10	-0.01	0.03
Egypt	0.00	-0.06	-0.09	-0.10	0.00	0.07	0.09	0.12	0.12
Hungary	-0.36	-0.37	-0.38	-0.38	-0.44	-0.35	-0.15	-0.04	-0.01
Indonesia	-0.28	-0.34	-0.33	-0.24	-0.09	0.04	0.12	0.20	0.24
Malaysia	-0.10	-0.20	-0.38	-0.53	-0.49	-0.34	-0.09	0.29	0.45
Mexico	-0.24	-0.25	-0.33	-0.28	-0.06	0.10	0.13	0.12	0.12
Peru	-0.23	-0.25	-0.17	-0.18	-0.05	-0.05	0.08	0.14	0.18
Philippines	-0.09	-0.12	-0.19	-0.23	-0.20	-0.07	0.06	0.16	0.17
Poland	0.13	0.04	0.03	0.02	-0.05	0.06	0.14	0.12	0.12
Russia	-0.06	-0.10	-0.23	-0.38	-0.44	-0.24	-0.22	-0.18	-0.16
South Africa	-0.15	-0.21	-0.21	-0.16	-0.06	0.02	0.11	0.18	0.19
Turkey	-0.26	-0.26	-0.17	-0.10	0.04	0.17	0.18	-0.11	-0.10
Ukraine	-0.34	-0.44	-0.53	-0.56	-0.43	-0.16	0.10	0.26	0.39
Uruguay	-0.29	-0.41	-0.59	-0.64	-0.61	-0.59	-0.45	-0.28	-0.08
Venezuela	-0.26	-0.27	-0.24	-0.14	-0.06	0.01	0.11	0.13	0.17
Korea	-0.28	-0.39	-0.58	-0.58	-0.29	-0.26	-0.13	0.13	0.32
El Salvador	-0.15	-0.12	-0.06	-0.01	0.01	0.02	0.03	0.06	0.08
Dominican Republic	-0.04	-0.08	-0.19	-0.18	-0.16	-0.09	-0.05	0.03	0.12
Average	-0.20	-0.24	-0.28	-0.27	-0.18	-0.09	-0.01	0.07	0.12
90th Percentile	-0.04	-0.09	-0.11	-0.07	0.01	0.07	0.13	0.19	0.31
10th Percentile	-0.35	-0.42	-0.51	-0.55	-0.44	-0.32	-0.15	-0.10	-0.07

A.2 Interest Rates, Volatility, and Market Sentiment

In this appendix we explain the decomposition we rely on to argue that part of movement in the level and volatility of interest rates is driven by an external market sentiment factor, and can therefore be considered partly exogenous.

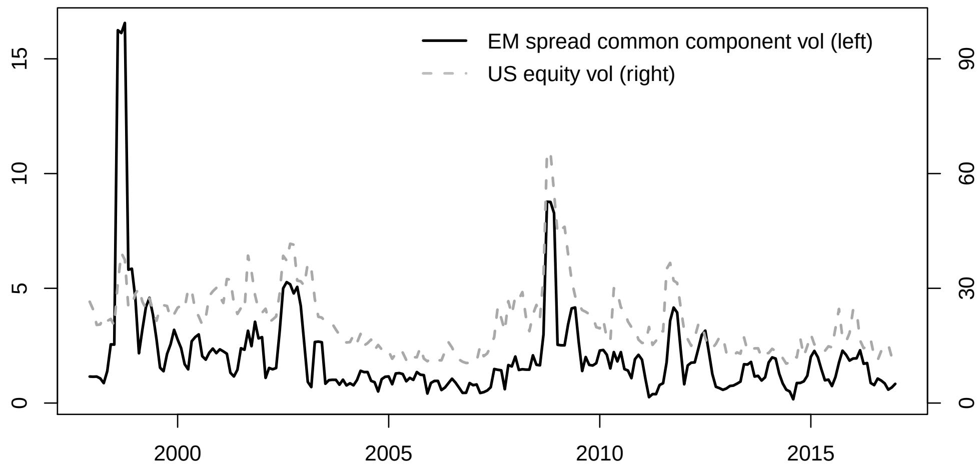
The interest rate at which a country borrows overseas can be decomposed into a baseline "risk-free" global interest rate—typically that of US government bonds—and a country-specific spread that reflects the risk premium charged by international investors. The risk-free component can be considered fully exogenous to any particular EME, as most of these countries are not large enough to unilaterally affect the international cost of borrowing. However, only a very small fraction of the volatility of interest rate in EMEs comes from the risk-free component. In fact, Fernández-Villaverde et al. (2011) estimate the stochastic volatility processes of both the risk-free and country spread components for four Latin American countries and find that the volatility of the risk-free rate is about an order of magnitude smaller than that of the country spread. In addition, they find a higher degree of time-variation in the country-spread volatility rather than in the risk-free rate volatility for all countries in their sample.

Regarding country spreads, the direction of causality can be twofold (see Uribe and Yue (2006) for a detailed discussion). On the one hand, Eichengreen and Mody (1998) show evidence that economic fundamentals are reflected on country spreads, especially on their cross-country levels. On the other hand, the authors also show that the time-variation of country spreads is largely unrelated to changes in economic fundamentals, and responds to a large extent to shifts in market sentiment. In that same vein, Uribe and Yue (2006) estimate a structural VAR for economic and financial variables on a panel of EMEs and find that 60 percent of the variation in country spreads is explained by the country spread shock itself (*i.e.*, independent of other "fundamental" shocks impacting the economy). More recently, Longstaff et al. (2011) show that there is a common underlying factor that drives 64 percent of the variation in country spreads in a sample of 26 emerging and developed countries. They use credit default swap (CDS) instead of bond spreads as the earlier literature does, but the interpretation of the results is equivalent.

As previously explained, we follow Longstaff et al. (2011) in identifying common latent factors in cross-country spreads using principal component analysis (PCA), but we run this on our EME bond data instead. We select a subsample of 13 countries that have long-enough country spread data to conduct this analysis, though we verify afterwards that results are qualitatively the same on the full country sample for the periods in which the data is balanced. We run the PCA on standardized monthly changes of the country spread only, since we already know that the risk-free rate component is a common factor across countries and it has relatively low degree of time-variability.

We find that the first principal component explains 57 percent of variation in country spreads in our sample and refer to it as the common market sentiment shock like the one described by Eichengreen and Mody (1998). The subsequent principal components explain a much smaller

Figure A.1: Rolling volatility of market sentiment factor versus US equity implied volatility



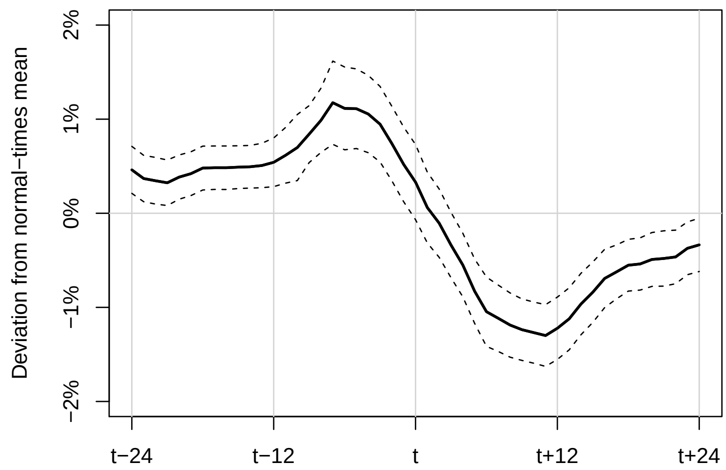
fraction of overall variability (less than 8% each individual one), and have a less straightforward interpretation.

We argue that the market sentiment factor we extract from EMEs country spreads does not uniquely reflect market sentiment regarding EMEs assets, but rather, that it captures broad financial market risk appetite. In Figure A.1, we overlay a gray dotted line of the implied volatility in S&P100 options (the VXO index, analogous to the VIX for the S&P500 index), which is typically considered to reflect broad market risk appetite. Whenever the VXO spikes up, the cost of hedging against moves in US equity prices increases, representing a higher degree of risk aversion. In our sample, the VXO is very highly correlated with the rolling volatility of the EME market sentiment factor. Most spikes tend to take place at the same time, and periods of steady low implied volatility in US equities (e.g., 2003-2007 and 2013-2015) tend to also display low realized volatility in the EME market sentiment factor. The overall correlation is 0.62, but it increases to 0.79 if we remove the 1998 episode that seems to be more centered on EMEs rather than broad risk assets.[1]

[1]The magnitude of the 1998 spike in the EME market factor volatility could also be due to lack of liquidity, as the EME bond market was not very developed at that point in time.

A.3 Output Gap around Sudden Stops

Figure A.2: Output gap (t denotes the initial month of a sudden stop)



B Social Planner's Recursive Problem

We consider a social planner that lacks commitment and that can only choose aggregate bond holdings for households but is still subject to the borrowing constraint. Following Klein et al. (2005), in order to solve for the time consistent policy, we focus on Markov stationary policy rules that only depend on the current state of the economy. In particular, they only depend on the aggregate state of the economy, (B, X). We solve for the constrained efficient allocation following the three steps described in Klein et al. (2005): (i) We first define a recursive competitive equilibrium for arbitrary policy rules; (ii) we then proceed to define a constrained-efficient allocation for arbitrary policy rules of future planners; and (iii) we define the constrained efficient allocation for the case in which such policies are time consistent, that is, we solve for the fixed point of the game being played by successive planners. In this problem, the social planner makes the borrowing decisions for the households, so the planner is the one facing the collateral constraint. Households are allowed to trade stocks of the tree freely without government intervention.

Let us consider a planner who chooses an arbitrary sequence of state-contingent lump-sum transfers, $\{T_t\}_{t=0}^{\infty}$. Given this sequence of transfers, we can write down the Bellman equation for the household's problem as follows:

$$V^A(s, T, X) = \max_{c,s'} \left\{ u(c) + \beta \mathbb{E}\left[V^A(s', T', X') \,|X \right] \right\} \tag{22}$$

subject to

$$c + \mathcal{Q}^A(T,X)\,s' = \left[\mathcal{Q}^A(T,X) + d(X)\right]s + T. \tag{23}$$

When solving this problem, the household takes the pricing function, $\mathcal{Q}^A(T,X)$, and the sequence of transfers as given. The solution to this problem is characterized by a policy rule for stock holdings, $s^A(s,T,X)$, such that Euler equation for stock holdings holds,

$$\mathcal{Q}^A(T,X) = \frac{\beta\mathbb{E}\left[u'(c')\left(\mathcal{Q}^A(T',X') + d(X')\right)|X\right]}{u'(c)}, \tag{24}$$

where

$$c + \mathcal{Q}^A(T,X)\,s^A(s,T,X) = \left[\mathcal{Q}^A(T,X) + d(X)\right]s + T. \tag{25}$$

Notice that the resource constraint of the economy implies that $T = B - \frac{B'}{1+r}$. Hence, given B, the planner actually chooses T by choosing B'. Therefore, we can rewrite the planner's policy rule as one that dictates B' as a function of the current aggregate state, (B,X). Call this policy rule $\Psi(B,X)$, and define the following functions:

$$\begin{aligned}
\mathcal{Q}(B,X) &\equiv \mathcal{Q}^A\left(B - \frac{\Psi(B,X)}{1+r}, X\right), \\
s(s,B,X) &\equiv s^A\left(s, B - \frac{\Psi(B,X)}{1+r}, X\right), \text{ and} \\
V(s,B,X) &\equiv V^A\left(s, B - \frac{\Psi(B,X)}{1+r}, X\right).
\end{aligned}$$

We can rewrite the optimality conditions for the household's problem as follows:

$$\mathcal{Q}(B,X) = \frac{\beta\mathbb{E}\left[u'(c')\left(\mathcal{Q}(B',X') + d(X')\right)|X\right]}{u'(c)}, \tag{26}$$

where

$$c + \mathcal{Q}(B,X)\,\hat{s}(s,T,X) = \left[\mathcal{Q}(B,X) + d(X)\right]s + B - \frac{\Psi(B,X)}{1+r}. \tag{27}$$

We can now define a recursive competitive equilibrium for an arbitrary policy rule $\Psi(B,X)$.

Definition 2 *A recursive competitive equilibrium for an arbitrary policy rule $\Psi(B,X)$ consists of a pricing function, $\hat{\mathcal{Q}}(B,X)$, and decision rules for households, $\hat{s}(s,B,X)$, with associated value function $\hat{V}(s,B,X)$, such that:*

1. *Given* $\Psi(B,X)$ *and* $\hat{\mathcal{Q}}(B,X)$*, households' decision rules* $\hat{s}(s,B,X)$ *and the associated value function* $\hat{V}(s,B,X)$ *solve the recursive problem of the household.*

2. *Markets clear:* $\hat{\mathcal{Q}}(B,X)$ *is such that* $\hat{s}(s,B,X)=1$ *and the resource constraint holds:* $c+\frac{B'}{1+r}=B+d(X)$*, where* $B'=\Psi(B,X)$*.*

The definition of such an equilibrium implies that the following set of equations must be satisfied:

$$\hat{\mathcal{Q}}(B,X) = \frac{\beta\mathbb{E}\left[u'\left(B'+d(X)-\frac{B''}{1+r}\right)\left[\hat{\mathcal{Q}}(B',X')+d(X')\right]|X\right]}{u'\left(B+d(X)-\frac{B'}{1+r}\right)}, \tag{28}$$

$$B' = \Psi(B,X), \text{ and} \tag{29}$$

$$B'' = \Psi(\Psi(B,X),X'). \tag{30}$$

Given that the planner we consider can only affect the allocation of bond holdings but cannot directly intervene in the markets for stocks, the pricing condition for $\hat{\mathcal{Q}}(B,X)$ has to hold in a constrained efficient allocation; in particular, this condition defines the price at which lenders value collateral in the current period borrowing constraint. Taking into account this kind of implementability constraint for the planner, we can now define the problem to be solved by a planner that takes as given the policy functions of future planners. Given future policy rules, $\Psi(B,X)$, associated pricing function $\hat{\mathcal{Q}}(B,X)$, and consumption rule $\mathcal{C}(B,X)$, the current planner chooses current consumption, c, and future bond holdings, B', to solve the following Bellman equation:

$$W(B,X)=\max_{c,B'}\{u(c)+\beta\mathbb{E}[W(B',X')|X]\} \tag{31}$$

subject to

$$c+\frac{B'}{1+r} = d(X)+B, \tag{32}$$

$$-\frac{B'}{1+r} \leq \kappa\tilde{\mathcal{Q}}(c,B',X), \tag{33}$$

where

$$\tilde{\mathcal{Q}}(c,B',X) = \frac{\beta\mathbb{E}\left[u'\left(\mathcal{C}(B',X')\right)\left(\mathcal{Q}(B',X')+d(X')\right)|X\right]}{u'(c)}, \text{ and} \tag{34}$$

$$\mathcal{C}(B',X') = d(X')+B'-\frac{\Psi(B',X')}{1+r'}. \tag{35}$$

Definition 3 *A constrained efficient allocation given a policy rule for future planners* $\Psi(B,X)$*, with associated pricing function* $\hat{\mathcal{Q}}(B,X)$ *and consumption rule* $\mathcal{C}(B,X)$*, consists of an optimal* *policy rule, $\hat{\Psi}(B,X)$, such that given functions $\Psi(B,X)$, $\hat{\mathcal{Q}}(B,X)$ and $\mathcal{C}(B,X)$, the current policy rule $B' = \hat{\Psi}(B,X)$ and associated value function $\hat{W}(B,X)$ solve the recursive problem of the current planner.*

By substituting (32) and (35) into (34), we can define the pricing function in terms of present and future aggregate bond holdings, (B, B'), as follows

$$\bar{\mathcal{Q}}(B,B',X) = \beta\mathbb{E}\left[\left.\frac{u'\left(B' + d(X') - \frac{\Psi(B',X')}{1+r'}\right)\left[\hat{\mathcal{Q}}(B',X') + d(X')\right]}{u'\left(d(X) + B - \frac{B'}{1+r}\right)}\right| X\right]. \tag{36}$$

Then, substituting (36) into (33), the first order conditions for the recursive problem of the current planner imply that $\hat{\Psi}(B,X)$ has to be such that the generalized Euler equation holds:

$$\begin{aligned} u'(\hat{\mathcal{C}}(B,X)) \;&-\; \hat{\mu}(B,X)\left[1 + \kappa(1+r)\xi(B,X)\right] \\ &= \;(1+r)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\hat{\mu}(B',X')\psi(B',X')|X\right], \end{aligned} \tag{37}$$

where $\psi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B}$, $\xi(B,X) = \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B'}$, and $\hat{\mathcal{C}}(B,X) = B + d(X) - \frac{\hat{\Psi}(B,X)}{1+r}$. The multiplier on the collateral constraint is given by

$$\begin{aligned} \hat{\mu}(B,X) \;&=\; \max\left\{0, \frac{1}{1+\kappa(1+r)\xi(B,X)}\left[u'\left(B + d(X) - \frac{\hat{\Psi}(B,X)}{1+r}\right)\right.\right. \\ &-\; \left.\left.\beta(1+r)\,\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\hat{\mu}(B',X')\psi(B',X')|X\right]\right]\right\}, \end{aligned} \tag{38}$$

where $\hat{\Psi}(B,X) = -(1+r)\kappa\bar{\mathcal{Q}}(B,\Psi(B,X),X)$.

Hence, we obtain that the functions that solve the planner's problem, $c = \hat{\mathcal{C}}(B,X)$ and $B' = \hat{\Psi}(B,X)$, must satisfy the following condition:

$$\begin{aligned} u'(\hat{\mathcal{C}}(B,X)) \;&-\; \hat{\mu}(B,X)\left[1 + \kappa(1+r)\xi(B,X)\right] \\ &= \;(1+r)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) + \kappa\hat{\mu}(B',X')\psi(B',X')\right], \end{aligned} \tag{39}$$

where

$$\psi(B,X) \;=\; \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B}, \tag{40}$$

$$\xi(B,X) \;=\; \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B'}, \text{ and} \tag{41}$$

$$\mathcal{C}(B,X) \;=\; B + d(X) - \frac{\Psi(B,X)}{1+r}. \tag{42}$$

Given the characterization of the allocation, we can now define a recursive constrained efficient allocation.

Definition 4 *The recursive constrained efficient allocation consists of functions* $\Psi(B,X)$, $\hat{\mathcal{Q}}(B,X)$, $\mathcal{C}(B,X)$, *and* $\hat{\Psi}(B,X)$ *with associated value function,* $\hat{W}(B,X)$, *such that*

1. $\hat{\mathcal{Q}}(B,X)$, $\mathcal{C}(B,X)$, $\hat{\Psi}(B,X)$, *and the associated value function* $\hat{W}(B,X)$, *constitute a constrained efficient allocation, given a policy rule for future planners,* $\Psi(B,X)$.

2. *The planner's plans are time-consistent:* $\hat{\Psi}(B,X) = \Psi(B,X)$ *and* $\bar{\mathcal{Q}}(B,\hat{\Psi}(B,X),X) = \hat{\mathcal{Q}}(B,X)$.

B.1 Derivation of equations (15) and (19)

Non-binding current collateral constraint: $\mu(B,X)=0$ Given our definition of $\bar{\mathcal{Q}}(B,B',X)$, notice that

$$\begin{aligned}\frac{\partial\bar{\mathcal{Q}}(B,B',X)}{\partial B} &= \beta\mathbb{E}\left\{-\frac{u'(\mathcal{C}(B',X'))\left(\hat{\mathcal{Q}}(B',X')+d(X')\right)}{u'(c)}\frac{u''(c)}{u'(c)}\right\}\\ &= -\frac{u''(c)}{u'(c)}\bar{\mathcal{Q}}(B,B',X),\end{aligned}$$

which implies that

$$\psi(B,X) = -\frac{u''(\mathcal{C}(B,X))}{u'(\mathcal{C}(B,X))}\hat{\mathcal{Q}}(B,X).$$

Therefore, when $\mu(B,X)=0$, condition (37) becomes an Euler equation with one wedge, μ,

$$u'\left(\hat{\mathcal{C}}(B,X)\right) = (1+r)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X')) - \kappa\hat{\mu}(B',X')\frac{u''(\mathcal{C}(B',X'))}{u'(\mathcal{C}(B',X'))}\hat{\mathcal{Q}}(B',X')\,|X\right],$$

precisely as in equation (19).

Binding current collateral constraint: $\mu(B,X)>0$ First note that the current planner has to choose B' subject to the collateral constraint

$$\frac{B'}{1+r}+\kappa\bar{\mathcal{Q}}(B,B',X)\geq 0. \tag{43}$$

Following Jeanne and Korinek (2018), note that if the left-hand side of the previous inequality is strictly increasing in B', then, for any given B, there is a unique B' such that this equation holds with equality. Call this $\bar{B}'$. Hence, for every (B,X), there exists a $\bar{B}'$ such that $\frac{\bar{B}'}{(1+r)}+\kappa\bar{\mathcal{Q}}(B,\bar{B}',X)=0$. Hence, when the collateral constraint is binding, the optimal policy rule by the current planner must be such that $\hat{\Psi}(B,X) = \bar{B}'$, and we obtain that $\frac{\hat{\Psi}(B,X)}{1+r} + \kappa\bar{\mathcal{Q}}(B,\hat{\Psi}(B,X),X) = 0$. Hence, this policy rule is unique, implying that $\xi(B,X) = 0$.[2]

Now, notice that the left-hand side of (43) is strictly increasing whenever

$$\frac{\partial}{\partial B'}\left(\frac{B'}{1+r} + \kappa\bar{\mathcal{Q}}(B,B',X)\right) = \frac{1}{1+r} + \kappa\xi(B,X) > 0 \tag{44}$$

which is precisely our assumption in footnote 7, $1+\kappa(1+r)\xi(B,X) > 0$.

In equilibrium, $\xi(B,X) < 0$, therefore we expect this condition to hold whenever κ is a small number.[3] Given the definition of $\bar{Q}(B,B',X)$, notice that

$$\frac{\partial\bar{\mathcal{Q}}(B,B',X)}{\partial B'} = \frac{\beta\mathbb{E}[\Omega(B,B',X)]}{u'(c)} + \frac{u''(c)}{u'(c)}\frac{\bar{\mathcal{Q}}(B,B',X)}{1+r} \tag{45}$$

where

$$\Omega(B,B',X) = u''(\mathcal{C}(B',X'))\frac{\partial\mathcal{C}(B',X')}{\partial B}[\mathcal{Q}(B',X') + d(X')] + u'(\mathcal{C}(B',X'))\frac{\partial\mathcal{Q}(B',X')}{\partial B}.$$

This last expression shows how the current planner takes into account how decisions affect future planners' actions by changing B'.

C Numerical Solution Method

We use a grid of 500 points for household savings, placing 80 percent of them around the region where the borrowing constraint binds in order to better capture the nonlinearities of the model. We truncate the grids in order to include 95 percent of the probability mass of shocks at the ergodic distribution, which was approximated by simulating the VAR for 1 million periods. To solve the system of rational expectations with occasionally binding constraints, we use an adaptation of the endogenous grid method of Carroll (2006). This appendix describes in detail our algorithm.

C.1 Competitive equilibrium

Let us denote by B the aggregate equilibrium savings of the economy, and by $X = (z,r,\sigma^r)$ the realization of exogenous shocks. We wish to find functions $\mathcal{B}(B,X)$, $\mathcal{C}(B,X)$, $\mathcal{Q}(B,X)$ and

[2] Another way to show this is by substituting the pricing function $\mathcal{Q}$ for the actual values $\bar{q} = \bar{\mathcal{Q}}(B,\bar{B}',X)$, which would imply that the constraint no longer depends on B'.

[3] Notice that when κ is small enough, the term $\kappa\mu(B',X')\psi(B',X')$ also becomes very small and $\hat{\Psi}(B,X)$ is unique in the case in which $\mu(B,X) = 0$.

$\mu(B, X)$ that satisfy

$$
\begin{aligned}
u'(\mathcal{C}(B,X)) &= \beta(1+r)\mathbb{E}\left[u'(\mathcal{C}(\mathcal{B}(B,X),X'))|X\right] + \mu(B,X), && (46)\\
\mathcal{C}(B,X) + \frac{\mathcal{B}(B,X)}{1+r} &= d(X) + B, && (47)\\
-\frac{\mathcal{B}(B,X)}{1+r} &\leq \kappa \mathcal{Q}(B,X), && (48)\\
\mathcal{Q}(B,X) &= \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}(\mathcal{B}(B,X),X'))\left[\mathcal{Q}(\mathcal{B}(B,X),X') + d(X')\right]}{u'(\mathcal{C}(B,X)) - \kappa\mu(B,X)}\right| X\right], && (49)\\
& && (50)
\end{aligned}
$$

We extend the endogenous grid method (EGM) of Carroll (2006) to our framework where there is a borrowing constraint that binds occasionally:

1. For each $\sigma^r \in \{\sigma^r_L, \sigma^r_H\} \equiv \mathcal{S}$, calculate the transition matrix for a discrete approximation to the VAR(1) process of (z, r) over $\mathcal{Z} \times \mathcal{R}$, with $\mathcal{Z} = \{z_1, \ldots, z_{Nz}\}$ and $\mathcal{R} = \{r_1, \ldots, r_{Nz}\}$.

2. Generate a grid $\bar{\mathcal{B}} = \{b_1, b_2, \ldots, b_N\}$, and an extended grid

$$\bar{\bar{\mathcal{B}}} = \bar{\mathcal{B}} \cup \{b_{N+1}, b_{N+2}, \ldots, b_{N+M}\},$$

where b_{N+M} is chosen such that the resulting $\max_X \mathcal{B}(b_N, X) \leq b_{N+M}$ (to be verified in the end).

3. Guess functions $\mathcal{C}_1(B, X)$, $\mathcal{Q}_1(B, X)$ and $\mathcal{Q}^c_1(B, X)$, for every $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$. The initial guess we use is

$$
\begin{aligned}
\mathcal{C}_1(B,X) &= d(X) + B\left(1 - \frac{1}{1+r}\right),\\
\mathcal{Q}_1(B,X) &= \frac{\beta}{1-\beta} d(X),
\end{aligned}
$$

which corresponds to the assumption that $\mathcal{B}(B, X) = B$, $z' = z$ and $r' = r$ for all (B, X).

4. Set $\mathcal{C}_0(B, X) = \mathcal{C}_1(B, X)$ and $\mathcal{Q}_0(B, X) = \mathcal{Q}_1(B, X)$ for each $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$.

5. Assume that (48) does not bind. Use (46) and (47) to calculate

$$
\begin{aligned}
\hat{\mathcal{C}}(B',X) &= u'^{-1}\left(\beta(1+r)\mathbb{E}\left[u'(\mathcal{C}_0(B',X'))|X\right]\right),\\
\hat{\mathcal{B}}(B',X) &= \hat{\mathcal{C}}(B',X) + \frac{B'}{1+r} - d(X).
\end{aligned}
$$

Notice that $\hat{\mathcal{B}}$ is the level of contemporaneous savings that yield an optimal savings decision B' when the realization of shocks is X and the borrowing constraint does not bind.

6. For each X, let us denote by $\hat{\bar{\bar{\mathcal{B}}}}(X)$ the endogenous grid of points generated by $\hat{\mathcal{B}}(B', X)$. For every X, interpolate B' from $\hat{\mathcal{B}}(B', X)$ to $\bar{\mathcal{B}}$, and denote the resulting function $\check{B}(B, X)$.

7. Calculate $\tilde{\mathcal{B}}(B, X) = \max\{\check{B}(B, X), -\kappa(1+r)\mathcal{Q}_0(B, X)\}$, and the corresponding consumption:
$$\tilde{\mathcal{C}}(B, X) = d(X) + B - \frac{\tilde{\mathcal{B}}'(B, X)}{1+r}.$$

8. Find $\mathcal{B}^*(B, X) = \min\{B \in \bar{\bar{\mathcal{B}}} : B \geq \tilde{\mathcal{B}}(B, X)\}$. Using (46) and (49) find
$$\tilde{\mu}(B, X) = u'(\tilde{\mathcal{C}}(B, X)) - \beta(1+r)\mathbb{E}\left[u'(\mathcal{C}_0(\mathcal{B}^*(B, X), X'))|X\right],$$
$$\tilde{\mathcal{Q}}(B, X) = \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}_0(\mathcal{B}^*(B, X), X'))\left[\mathcal{Q}_0(\mathcal{B}^*(B, X), X') + d(X')\right]}{u'(\tilde{\mathcal{C}}(B, X)) - \kappa\tilde{\mu}(B, X)}\right| X\right],$$

9. For every $(B, X) \in \bar{\mathcal{B}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$, update
$$\mathcal{C}_1(B, X) = \alpha\tilde{\mathcal{C}}(B, X) + (1-\alpha)\mathcal{C}_0(B, X),$$
$$\mathcal{Q}_1(B, X) = \alpha\tilde{\mathcal{Q}}(B, X) + (1-\alpha)\mathcal{Q}_0(B, X)$$

   for some $\alpha \in (0, 1]$. For $B \in \bar{\bar{\mathcal{B}}} \backslash \bar{\mathcal{B}}$, set $\mathcal{C}_1(B, X) = \mathcal{C}_1(b_N, X)$ and $\mathcal{Q}_1(B, X) = \mathcal{Q}_1(b_N, X)$.

10. Repeat steps 4-9 until convergence.

C.2 Constrained efficient allocation

The constrained efficient allocation satisfies

$$u'(\mathcal{C}(B, X)) - \mu(B, X)\left[1 + \kappa(1+r)\xi(B, X)\right] \tag{51}$$
$$= (1+r)\beta\mathbb{E}\left[u'(\mathcal{C}(B', X')) + \kappa\mu(B', X')\psi(B', X')|X\right],$$
$$\mathcal{Q}(B, X) = \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}(\mathcal{B}(B, X), X'))\left[\mathcal{Q}(\mathcal{B}(B, X), X') + d(X')\right]}{u'(\mathcal{C}(B, X)) - \kappa\mu(B, X)}\right| X\right], \tag{52}$$

together with (47) and (48). Some steps of the EGM algorithm change with respect to the solution of the competitive equilibrium:

3. Guess functions $\mathcal{C}_1(B, X)$, $\mathcal{Q}_1(B, X)$ and $\mu_1(B, X)$ for every $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$. The initial guess we use is: $\mu_1(B, X) = 0$.

4. Set $\mathcal{C}_0(B, X) = \mathcal{C}_1(B, X)$, $\mathcal{Q}_0(B, X) = \mathcal{Q}_1(B, X)$ and $\mu_0(B, X) = \mu_1(B, X)$ for each $(B, X) \in \bar{\bar{\mathcal{B}}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$.

Calculate:

$$\psi(B,X) = -\frac{u''(\mathcal{C}_0(B,X))}{u'(\mathcal{C}_0(B,X))}\mathcal{Q}_0(B,X).$$

Use the numerical derivatives of C_0 and Q_0 with respect to B to calculate $\xi(B,X)$ using equation (45) of Appendix B.

5. Assume that (48) does not bind. Use (51) and (47) to calculate:

$$\hat{\mathcal{C}}(B',X) = u'^{-1}\left(\beta(1+r)\mathbb{E}\left[u'(\mathcal{C}_0(B',X')) + \kappa\mu_0(B',X')\psi(B',X')|X\right]\right),$$
$$\hat{\mathcal{B}}(B',X) = \hat{\mathcal{C}}(B',X) + \frac{B'}{1+r} - d(X).$$

8. Find $\mathcal{B}^*(B,X) = \min\{B \in \bar{\bar{\mathcal{B}}} : B \geq \tilde{\mathcal{B}}(B,X)\}$. Using (51) and (52), find:

$$\tilde{\mu}(B,X) = \frac{1}{1+\kappa(1+r)\xi(B,X)}\Big\{u'(\tilde{\mathcal{C}}(B,X))$$
$$-\beta(1+r)\mathbb{E}\left[u'(\mathcal{C}_0(\mathcal{B}^*(B,X),X')) + \kappa\mu_0(\mathcal{B}^*(B,X),X')\psi(\mathcal{B}^*(B,X),X')|X\right]\Big\},$$
$$\tilde{\mathcal{Q}}(B,X) = \beta\mathbb{E}\left[\left.\frac{u'(\mathcal{C}_0(\mathcal{B}^*(B,X),X'))\left[\mathcal{Q}_0(\mathcal{B}^*(B,X),X') + d(X')\right]}{u'(\tilde{\mathcal{C}}(B,X)) - \kappa\tilde{\mu}(B,X)}\right| X\right],$$

9. For every $(B,X) \in \bar{\mathcal{B}} \times \mathcal{Z} \times \mathcal{R} \times \mathcal{S}$, update:

$$\mathcal{C}_1(B,X) = \alpha\tilde{\mathcal{C}}(B,X) + (1-\alpha)\mathcal{C}_0(B,X),$$
$$\mathcal{Q}_1(B,X) = \alpha\tilde{\mathcal{Q}}(B,X) + (1-\alpha)\mathcal{Q}_0(B,X)$$
$$\mu_1(B,X) = \alpha\tilde{\mu}(B,X) + (1-\alpha)\mu_0(B,X).$$

for some $\alpha \in (0,1]$. For $B \in \bar{\bar{\mathcal{B}}}\backslash\bar{\mathcal{B}}$, set $\mathcal{C}_1(B,X) = \mathcal{C}_1(b_N,X)$, $\mathcal{Q}_1(B,X) = \mathcal{Q}_1(b_N,X)$ and $\mu_1(B,X) = \mu_1(b_N,X)$.

10. Repeat steps 4-9 until convergence.

D Three-Period Model

In this appendix we consider a simple three-period version of our model to focus on understanding how a increasing mean preserving spreads in interest rates can generate an increase in borrowing depending on the current level of interest rates.

Consider a three period model, $t = 1,2,3$, and let $R_t \equiv 1+r_t$. Household i chooses $\{c_{i,t}\}_{t=1,2,3}$

and $\{b_{i,t+1}, s_{i,t+1}\}_{t=1,2}$ to maximize

$$u(c_{i,1}) + \beta u(c_{i,2}) + \beta^2 u(c_{i,3})$$

subject to

$$\begin{aligned} c_{i,1} + q_1 s_{i,2} + \frac{b_{i,2}}{R_1} &= (1-\alpha) d_1 + (q_1 + \alpha d_1) s_{i,1} + b_{i,1} \\ c_{i,2} + q_2 s_{i,3} + \frac{b_{i,3}}{R_2} &= (1-\alpha) d_2 + (q_2 + \alpha d_2) s_{i,2} + b_{i,2} \\ c_{i,3} &= (1-\alpha) d_3 + \alpha d_3 s_{i,3} + b_{i,3} \end{aligned}$$

and

$$\begin{aligned} -\frac{b_{i,2}}{R_1} &\leq \kappa q_1 s_{i,2} \\ -\frac{b_{i,3}}{R_2} &\leq \kappa q_2 s_{i,3} \end{aligned}$$

where $d_1 = d_2 = d_3 = 1$, $R_1 = \bar{R}$ and $R_2 = \bar{R} + x$ with probability $\frac{1}{2}$ and $R_2 = \bar{R} - x$ with equal probability, where $x > 0$. Hence, $\mathbb{E}[R_2] = \bar{R}$ and $\mathbb{V}[R_2] = x^2$, and an increase in x represents a mean preserving spread for the interest rate in period 2. Assume that $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

Period 3 At $t = 3$, aggregate consumption is given by $c_3 = d_3 + b_3 = 1 + b_3$.

Period 2 At $t = 2$, household i chooses $c_{i,2}, b_{i,3}$ and $s_{i,3}$ such that

$$\begin{aligned} c_{i,2}^{-\gamma} - \mu_{i,2} &= \beta R_2 c_{i,3}^{-\gamma} \\ q_2 &= \alpha \frac{c_{i,3}^{-\gamma}}{c_{i,2}^{-\gamma}} \end{aligned}$$

where $\mu_{i,2} = 0$ if the collateral constraint is not binding. In order to simplify our analysis, let us assume that parameter values are such that the collateral constraint is not binding in $t = 1, 2$.

If the collateral constraint is not binding in $t = 2$, then aggregate consumption and b_3 are determined by the following two equations

$$\begin{aligned} c_2 &= (\beta R_2)^{-\frac{1}{\gamma}} (1 + b_3) \\ c_2 &= 1 + b_2 - \frac{b_3}{R_2}. \end{aligned}$$

Let $w_t \equiv 1 + b_t$ denote wealth in period $t = 1, 2, 3$. Solving for c_2 as a function of w_2 we obtain that

$$c_2 = w_2 - \frac{1}{R_2}\frac{(\beta R_2)^{1-\frac{1}{\gamma}} + \beta R_2 w_2}{(\beta R_2)^{1-\frac{1}{\gamma}} + \beta}.$$

and q_2 is simply given by $q_2 = \alpha\left(\frac{1+b_3}{c_2}\right)^{-\gamma}$. Notice that that c_2 depends on interaction between w_2 and R_2. Moreover, w_2 is in turn a function of R_1.

Period 1 At $t = 1$, household i chooses $c_{i,1}, b_{i,2}$ and $s_{i,2}$ to satisfy

$$1 = \beta \bar{R} \frac{\mathbb{E}\left[c_{i,2}^{-\gamma}\right]}{c_{i,1}^{-\gamma}} \tag{53}$$

and asset prices are simply given by $q_1 = \frac{c_{i,2}^{-\gamma}(q_2+\alpha)}{c_{i,1}^{-\gamma}}$.

The intuition typically used to explain the effects of an increase in the volatility of interest rates goes as follows. Consider the first equation. If R_2 is more volatile, then c_2 is more volatile and by Jensen's inequality it must be the case that $\mathbb{E}\left[c_{i,2}^{-\gamma}\right]$ increases, and therefore $c_{i,1}$ must decrease given that $\bar{R}$ is fixed. Hence, households increase their savings (Fernández-Villaverde et al., 2011). However, notice that this logic—in particular Jensen's inequality—only holds as long as $\mathbb{E}\left[c_{i,2}\right]$ stays relatively constant for the different volatilities of interest rates. If somehow, $\mathbb{E}\left[c_{i,2}^{-\gamma}\right]$ decreases because $\mathbb{E}\left[c_{i,2}\right]$ increases, then this result does no longer hold.

Now consider

$$\begin{aligned}\mathbb{E}\left[c_{i,2}^{-\gamma}\right] = &\frac{1}{2}\left(w_2 - \frac{1}{\bar{R}+x}\frac{\left(\beta(\bar{R}+x)\right)^{1-\frac{1}{\gamma}} + \beta(\bar{R}+x)w_2}{\left(\beta(\bar{R}+x)\right)^{1-\frac{1}{\gamma}} + \beta}\right)^{-\gamma} \\ &+ \frac{1}{2}\left(w_2 - \frac{1}{\bar{R}-x}\frac{\left(\beta(\bar{R}-x)\right)^{1-\frac{1}{\gamma}} + \beta(\bar{R}-x)w_2}{\left(\beta(\bar{R}-x)\right)^{1-\frac{1}{\gamma}} + \beta}\right)^{-\gamma}.\end{aligned}$$

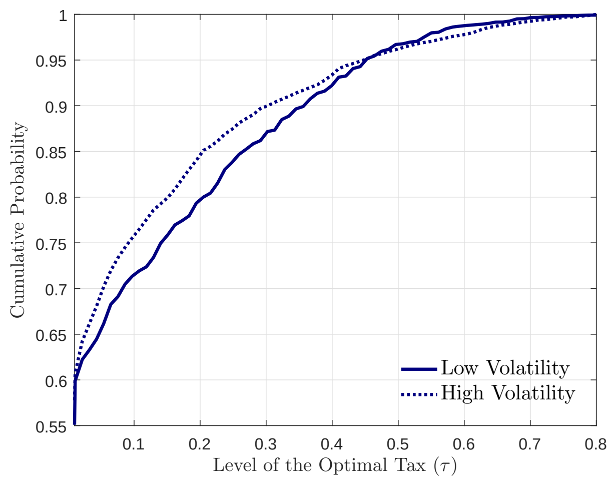
Differentiating the previous expected value with respect to x, it can be shown that the sign of $\frac{\partial}{\partial x}\mathbb{E}\left[c_{i,2}^{-\gamma}\right]$ depends on the value of w_2. For instance, for $\gamma = 2$, $\beta = 0.96$, $\bar{R} = \beta^{-1}$, one obtains that

$$\frac{\partial}{\partial x}\mathbb{E}\left[c_{i,2}^{-\gamma}\right] < 0 \text{ for } w_2 = 0.8 \text{ and}$$
$$\frac{\partial}{\partial x}\mathbb{E}\left[c_{i,2}^{-\gamma}\right] > 0 \text{ for } w_2 = 0.1.$$

Lastly, notice that if the substitution effect dominates the wealth effects in period 1, then $\frac{\partial w_2}{\partial R_1} > 0$. Hence, higher interest rates in period 1 can generate enough wealth in $t = 2$ such that households actually decide to adjust and decrease savings for higher levels of volatility of interest rates as we discussed in section 4: saving more is not a good vehicle to insure against risks and bonds become riskier as an asset.

E CDF of Optimal Taxes under High and Low Volatility

Figure E.3: Cumulative Distribution Function of Optimal Taxes



- Statistics derived from model simulations:
  - Probability of zero tax given low volatility: 42.65%
  - Probability of zero tax given high volatility: 47.90%
  - Average tax (excluding zeros) given low volatility: 16.47%
  - Average tax (excluding zeros) given high volatility: 15.78%