Macroprudential Policy in the Presence of External Risks*

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Abstract

We document the relevance of external risks—shocks to the level and volatility of world interest rates—in shaping economic activity in emerging market economies (EMEs) and characterize optimal macroprudential policy in response to these risks in a small open economy subject to financial crises. Low and stable interest rates reinforce overborrowing arising from a pecuniary externality generated by collateral constraints that depend on asset prices. We show that this mechanism leads to greater exposure to crises typically accompanied by abrupt increases in interest rates and a persistent rise in their volatility, as observed during sudden stops in EMEs. A tax on international borrowing that implements a social planner's optimal policy can be decomposed into three factors: the severity of future crises, the ability of the planner to dampen future crises, and the incidence of future crises. We show that the effects of interest rate volatility on these factors implies that, qualitatively, the tax responds non-monotonically with respect to the direction of volatility shocks and that, quantitatively, increasing macroprudential taxes is seldom the optimal response to an increase in interest rate volatility.

JEL classification: E6, F3, F4

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1 Introduction

Global factors affect economic activity in emerging market economies (EMEs) independently of their specific macroeconomic conditions. The 2008 Global Financial Crisis not only provided evidence for these risks, but also raised the possibility of using macroprudential policies to reduce the size and frequency of crises associated with these forces (Miranda-Agrippino and Rey, 2015). A large body of literature has suggested that such external risks affect EMEs by generating abrupt changes in the level and, to a lesser extent, in the volatility of interest rates at which EMEs borrow in international financial markets (Neumeyer and Perri, 2005; Uribe and Yue, 2006; Fernández-Villaverde et al., 2011). However, despite the evident risks for EMEs associated with such changes, the importance of movements in the volatility of interest rates in shaping economic activity in EMEs has not been sufficiently documented, and the implications of these risks for the design and implementation of optimal macroprudential policy have not been carefully studied.

This paper aims to fill these gaps in the literature by, first, documenting the relevance of shocks to the volatility of interest rates for economic activity in a large sample of EMEs and, second, characterizing optimal macroprudential policy in response to external risks in the form of shocks to the level and volatility of interest rates in a model of financial crises. More specifically, in the first part of this paper we conduct an empirical analysis of movements in the level and volatility of interest rates at which EMEs borrow internationally. We study how these measures evolve over time, their comovement with output, and their behavior around sudden stops for a broad sample of EMEs. We then estimate a Markov regime-switching VAR process for output and interest rates in which the volatility of interest rates varies across two regimes. In the second part of the paper we extend a Fisherian model of macroprudential policy to include external risks. The quantitative framework we consider consists of a small open economy facing an external borrowing limit that depends on the value of a domestic non-tradable asset in the spirit of Jeanne and Korinek (2018) and Bianchi and Mendoza (2018). External risks arise from two sources: shocks to the level of interest rates, and the existence of multiple stochastic regimes in the variance of interest rates at which the economy borrows. We feed the stochastic process for output and interest rates into our model to compare its predictions with the data. We then solve for the macroprudential tax on international borrowing that implements the optimal allocations by a constrained social planner and analyze how this tax responds to shocks to the level and volatility of interest rates.

Our empirical exercise provides evidence that time variation in the level and volatility of interest rates is pervasive across time and countries. For our broad sample of EMEs, we also confirm that increases in the level and volatility of interest rates are negatively correlated with output (Neumeyer and Perri, 2005; Uribe and Yue, 2006; Fernández-Villaverde et al., 2011). Moreover, we establish that the majority of variation in interest rates, and therefore their volatility, is explained by a common factor across EMEs (Eichengreen and Mody, 1998; Longstaff et al., 2011), which can be interpreted as exogenous to countries' fundamentals. Our main empirical finding documents the anatomy of increases in the level and volatility of interest rates and of their common factor around sudden stops. We show that, during sudden stops, interest rates increase 1.7 percentage points and their volatility 0.3 percentage point and that these changes are significantly driven by movements in the exogenous common factor. Lastly, our estimation of the Markov regime-switching model confirms that there is a statistically significant difference in the volatility of interest rates across regimes. Therefore, in addition to confirming some key results in the literature on EME business cycles, we contribute to this literature with our last two findings.

In the second part of the paper we extend the model of Jeanne and Korinek (2018) to include shocks to the level and volatility of interest rates and calibrate it to match two moments in the data: (i) the increase in interest rates around sudden stops and (ii) the prevalence of sudden stops. After feeding into the model our estimated process for output and interest rates, we derive four main results. First, while high volatility of interest rates can lead private agents and the social planner to decrease leverage because of precautionary motives, when the level of interest rates is high these motives dissipate and higher volatility leads to more borrowing. The key to understanding this result lies in the fact that high initial levels of interest rates generate enough savings to insure against volatility shocks and bonds become less of an insurance vehicle and more of a traditional asset—with a low return for high volatility—implying that agents want to hold less of it and borrow more. Second, this complex interaction of forces implies that, qualitatively, the macroprudential tax on international borrowing does not respond monotonically to changes in volatility. For instance, while for high interest rates an increase in their volatility calls for higher taxes, this result does not hold for average levels of interest rates. To better understand the forces shaping the optimal tax function, we decompose it into three factors—(i) the severity of future crises, (ii) the ability of the planner to dampen future crises, and (iii) the incidence of future crises—and show that the effects of volatility on the incidence of future crises plays a crucial role.

Third, our simulated model generates quantitatively relevant responses to shocks to the level and volatility of interest rates by agents in the competitive equilibrium and by the social planner, and these responses and the model dynamics in general replicate our data around sudden stops.

Lastly, we show that our simulations imply that, contrary to conventional wisdom, an increase in macroprudential taxes on international borrowing in response to an increase in the volatility of interest rates is seldom optimal. Furthermore, there is a 50 percent probability that a mean preserving spread in interest rates calls for no change in optimal taxes whatsoever.

Our theoretical results contribute to the literature on Fisherian models of macroprudential policy in multiple respects (Korinek and Mendoza, 2013). First, relative to other work that has extended these models to include new types of shocks (Bianchi et al., 2012, 2016), we extend a model with asset prices to include shocks to the volatility of interest rates. Second, we provide a calibration of the simple model with asset prices of Jeanne and Korinek (2018) that can match the data around sudden stops relatively well. Lastly, we provide a novel decomposition of the macroprudential tax into the three factors previously mentioned and show how their interaction can result in the quantitative irrelevance of interest rate volatility for macroprudential policy.[1]

The rest of the paper is organized as follows. In Section 2 we conduct our empirical analysis and estimate the stochastic process that we feed into our theoretical framework. Section 3 introduces the model and describes the competitive equilibrium and the problem of the constrained social planner. In Section 4 we map the model to the data and present the results of our numerical exercises. We show that the dynamics of interest rates around episodes of sudden stops in the model are consistent with their empirical counterparts and explain how the optimal response of the planner is shaped by external shocks. Section 5 concludes.

2 Empirical Analysis: Interest Rates in EMEs and Sudden Stops

In this section, we document the empirical relevance of shocks to the level and volatility of interest rates for EMEs' economic activity. First, we document time variation in both the level and volatility of external interest rates for a broad sample of EMEs. Second, we show that output is negatively correlated with the level and volatility of interest rates and that a common factor in EME country spreads drives most of their monthly variation. This factor can be interpreted as a market sentiment factor exogenous to countries' fundamentals. In the third and main empirical result, we document that increases in the level and volatility of interest rates—in particular those driven by market sentiment—are a key feature of sudden stops in EMEs. Lastly, we estimate a regime-switching vector autoregressive (VAR) model for our sample of EMEs that captures the time-varying nature of external interest rate volatility. We use this regime-switching VAR as an input into the theoretical model we present in Section 3.

Even though some empirical regularities that we document here have been highlighted in

[1]We thank one anonymous referee for the suggestion to decompose the optimal tax into these three forces.

existing work, we contribute to the empirical literature on the effects of interest rates on EMEs in two particular ways. First, we document the interplay between interest rates, their volatility, and business cycles using a broader set of countries and longer time series than the existing literature, therefore confirming the robustness of previous findings (Eichengreen and Mody, 1998; Uribe and Yue, 2006; Neumeyer and Perri, 2005; Fernández-Villaverde et al., 2011; Longstaff et al., 2011). Second, we contribute by documenting increases in the level and volatility of a common factor driving interest rates around sudden stop episodes, which partially motivate our theoretical analysis in the remaining sections of this paper.

Data sources For our empirical analysis we consider a sample of 23 countries that have interest rate, output, and current account data available for a sufficiently long period of time.[2] Most of our analysis is carried out at a monthly frequency; however, we also present some of our analyses at quarterly or annual frequencies to compare results with the existing literature.

We compute external interest rates using J.P. Morgan's Emerging Market Bond Index Global (EMBIG) spread and a measure of US five-year real rates. The output gap is measured using seasonally adjusted quarterly GDP at constant prices, de-trended using a Hodrick-Prescott filter, as is common in the literature. Sudden stop episodes are identified using monthly trade balance and international reserve data based on the methodology introduced by Calvo et al. (2008) and extended by Reyes-Heroles and Tenorio (2019).

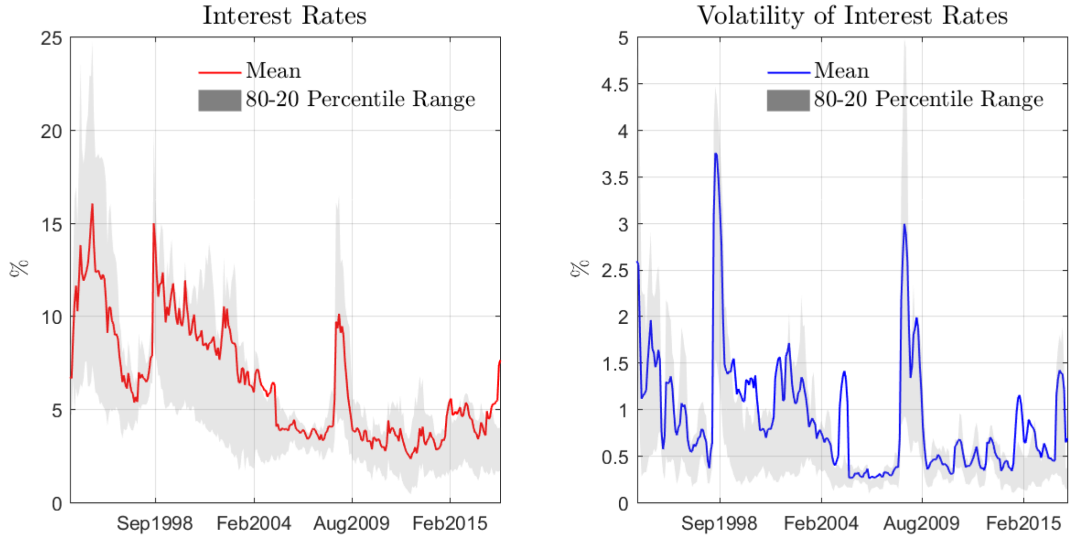
2.1 Interest Rates and their Volatility in EMEs

We start by analyzing the unconditional behavior of the level and volatility of interest rates in EMEs. Figure 1 shows the evolution of the mean and 80-20 cross-sectional percentile ranges of the level (left panel) and volatility (right panel) of interest rates for our sample of EMEs. The data in the figure clearly show the presence of shocks to the level and of regime-switches in the volatility of interest rates.

Columns (3) through (6) of Table 1 provide further evidence of time variation in the level and volatility of interest rates. On average, the 90-10 interquantile range (IQR) of interest rate volatility in our sample is 0.95, which is high compared with the average volatility across countries, at only 0.52 (columns (5) and (6)). The outliers here are Argentina, Ukraine, and Ecuador, all of which have experienced repeated volatility spikes largely because of idiosyncratic crises. However, we find evidence of time-varying volatility even in the spreads of countries with relatively strong economic fundamentals, such as Chile, Hungary, Malaysia, and Poland. Hence,

[2] This sample is shown in Table 1. Column (1) of the table indicates the time span of data available for each country.

Figure 1: Evolution of interest rates and their volatility



Note: For interest rates we consider J.P. Morgan's EMBIG (left panel). The measure of volatility shown in the right panel is the 7-month rolling standard deviation of the EMBIG.

we conclude that the interest rates at which EMEs borrow and their volatility show significant variation over time.

Column (2) of Table 1 also shows that the average standard deviation of the output gap across our sample of countries is 2.4 percent, which is in line with what has been documented in the literature on EMEs' business cycles.

2.2 Interest Rates, Volatility, and Output

Having provided evidence of time variation in the levels and volatility of interest rates in our sample of EMEs, we now explore the interplay of these measures with economic activity.

Figure 2 shows the cross-correlations of the output gap with interest rate levels (panel (a)) and with interest rate volatility (panel (b)). The figures show the average cross-correlation across the countries at different horizons, measured in quarters. The figures also display the 90-10 percentiles of the moment measured across our sample of countries. Tables A.1 and A.2 in Appendix A (available online) show the cross-correlations disaggregated by country level.

Consistent with the literature, panel (a) shows that interest rates have a negative correlation with the output gap in EMEs and that increases in interest rates tend to forecast declines in future output. The magnitudes of the cross-correlations are smaller (in absolute value) than the ones estimated by Neumeyer and Perri (2005) for five EMEs up to 2001, but both results go in the same direction. Panel (b) shows that the cross-correlation of interest rate volatility with

Table 1: Descriptive statistics

	Sample	Output Gap	Interest Rates Levels		Interest Rates Volatility	
		s.d.	Mean	IQR	Mean	IQR
	(1)	(2)	(3)	(4)	(5)	(6)
Argentina	Dec/93-Dec/17	4.62	14.74	44.55	1.64	3.15
Brazil	Apr/94-Dec/17	1.68	5.57	11.64	0.6	1.04
Bulgaria	Mar/95-Nov/13	2.43	5.45	14.25	0.52	0.87
Chile	May/99-Mar/17	1.63	1.21	3.98	0.20	0.34
Colombia	Feb/97-Feb/17	1.64	3.42	7.95	0.33	0.47
Ecuador	Feb/95-Sep/16	2.23	11.89	15.03	1.34	2.63
Egypt	Mar/02-Dec/13	2.06	2.05	3.59	0.31	0.67
Hungary	Jan/99-Mar/17	1.61	1.59	3.76	0.28	0.51
Indonesia	May/04-Mar/17	0.47	2.19	3.45	0.25	0.34
Malaysia	Oct/96-Mar/17	2.57	1.82	4.86	0.27	0.39
Mexico	Dec/93-Feb/17	2.32	3.64	7.80	0.38	0.65
Peru	Mar/97-Mar/17	1.66	3.26	7.75	0.37	0.68
Philippines	Dec/93-Mar/17	1.18	3.46	6.85	0.32	0.48
Poland	Jun/95-Sep/17	1.36	1.90	5.31	0.22	0.31
Russia	Dec/97-Sep/17	2.91	6.35	12.67	0.93	1.34
South Africa	Dec/94-Dec/16	1.15	2.56	4.92	0.27	0.50
Turkey	Jun/96-Mar/17	5.18	3.94	7.35	0.40	0.69
Ukraine	May/00-Dec/16	4.54	7.94	16.83	1.15	2.85
Uruguay	May/01-Sep/17	2.32	2.99	5.95	0.38	0.72
Venezuela	Mar/97-Dec/15	5.28	9.76	11.14	0.92	1.62
Korea	Dec/93-Mar/04	3.07	3.01	5.29	0.34	0.51
El Salvador	Apr/02-Sep/17	1.21	2.98	2.67	0.25	0.49
Dominican Republic	Mar/07-Sep/17	1.67	3.37	2.54	0.35	0.61
Average		2.38	4.57	9.14	0.52	0.95

Notes: Volatility is measured as the intra-quarter standard deviation (s.d.) of changes in monthly interest rates.

output is even more negative than for interest rate levels. The magnitude of these moments is within the range reported by Fernández-Villaverde et al. (2011) for a sample of four EMEs.

In order to account for the fact that interest rates at which EMEs borrow might not be entirely exogenous to countries' fundamentals, we follow Longstaff et al. (2011) and identify a common factor driving interest rates for a subsample of our countries.[3] We find that the first principal component explains 57 percent of the variability of interest rates in our sample, which is in line with the finding of Longstaff et al. (2011). Moreover, all country spreads in our sample have positive loadings on this factor, suggesting that there is a common market sentiment shock playing a key role in driving EME country spreads. Henceforth, we denote this first principal component as the *market sentiment factor*.

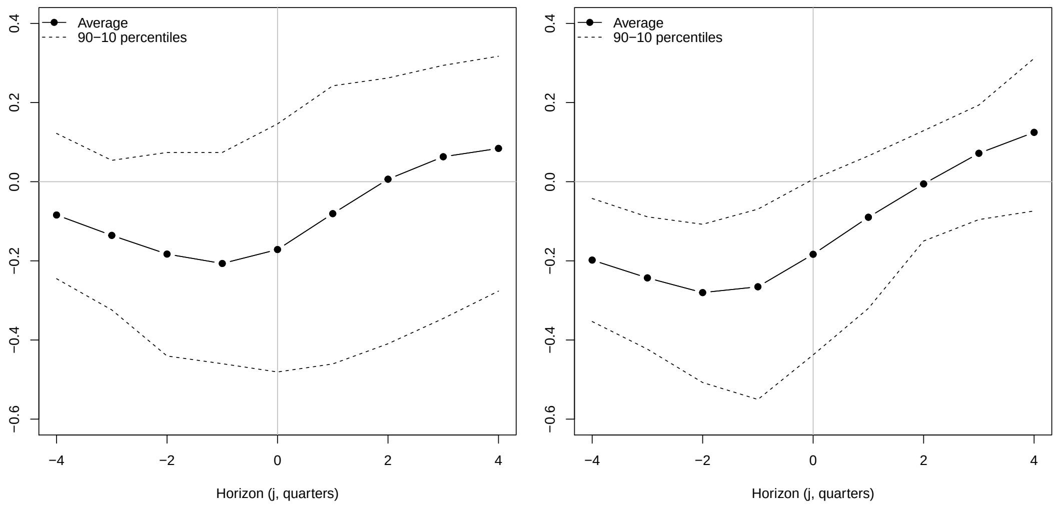
Notably, the market sentiment factor has a substantial degree of time-varying volatility itself. The dark line in Figure A.1 in Appendix A (available online) shows the three-month rolling volatility of the market sentiment factor across time. The data show clear spikes in periods when EME credit was under market stress: the 1998 Russian debt crisis; the 2001-2002 US

[3]We select a subsample of 13 countries that have long enough country spread data to conduct this analysis, though we verified afterward that the results are qualitatively the same for our full country sample for the periods in which the data are balanced. In online Appendix A we provide additional details on our factor decomposition.

Figure 2: Relationship between output and interest rate levels and volatility

(a) Cross-correlation of output gap and interest rate *level*

(b) Cross-correlation of output gap and interest rate *volatility*



market crash; the 2008 global financial crisis; and the 2011-2012 European debt crisis. Even though the 1998 event originated in a specific corner of the EME world, the fact that the common component in country spreads became so volatile at this point shows that overall market sentiment turned against EME credit, thus representing a partially exogenous shock for the rest of the countries in our sample. In Appendix A we also show that the market sentiment factor reflects not only market sentiment regarding EME assets, but also broad financial market risk appetite, thus reinforcing the exogeneity of this factor to EME fundamentals.

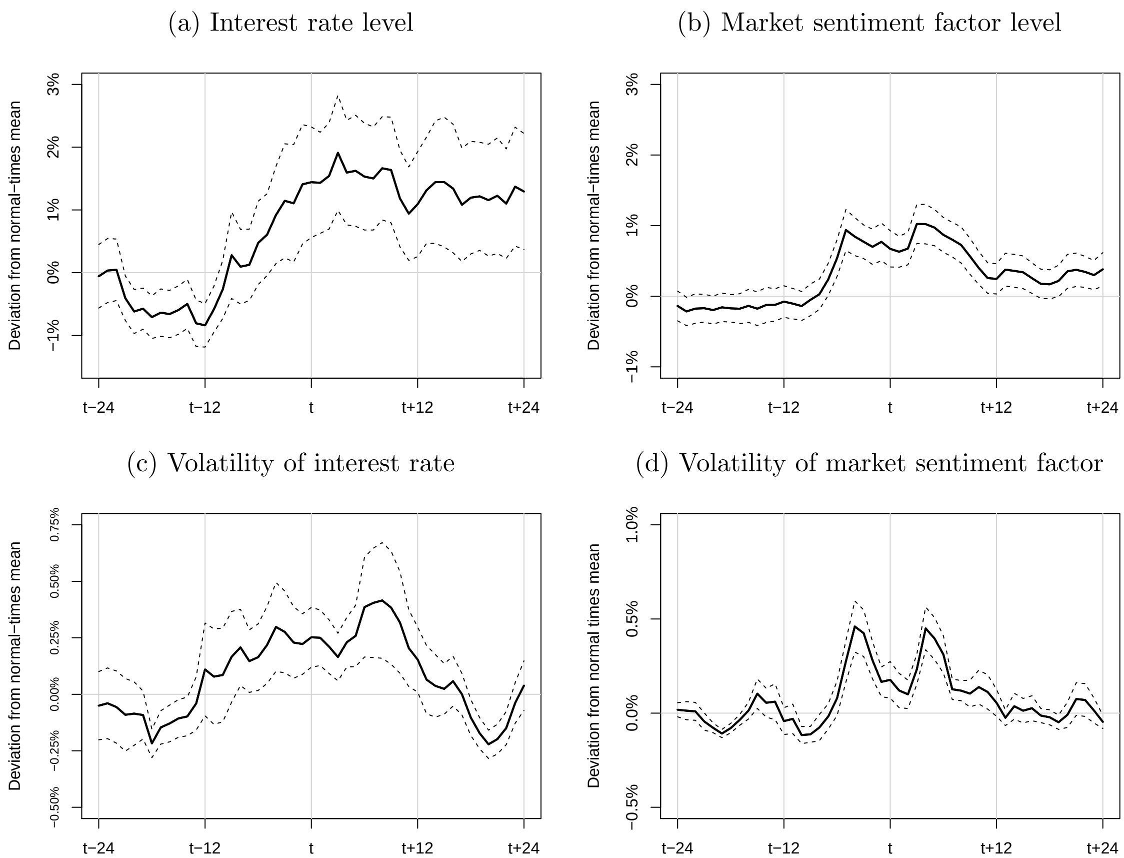
From our previous results we conclude that the level and volatility of interest rates comoves with output and that a majority of the variation in EME country spreads is driven by global factors that are exogenous to these economies' fundamentals.

2.3 Interest Rate, Volatility, and Sudden Stops

In this section we focus on sudden stops—events associated with large net capital outflows and output contractions—and study how the level and volatility of both interest rates and the market sentiment factor behave around these episodes. We do this by conducting an event-window analysis. In particular, we compare the average behavior of the interest rate and market sentiment levels and volatilities around sudden stops against their corresponding normal-times levels for the 73 sudden stop events classified by Reyes-Heroles and Tenorio (2019).[4]

[4]These 73 episodes are obtained by extending the analysis of Calvo et al. (2008). These episodes imply that sudden stops take place every 107 months, and they last for 9.9 months, on average. Countries in the sample

Figure 3: Event study around sudden stops: Interest rate and market sentiment factor level and volatility



Note: The graphs depict the deviations of variables from the normal-times country-specific mean, using all data available for our sample of EMEs. t denotes the month in which the sudden stop begins. Dotted lines represent one standard error intervals.

Figure 3 shows the results of our event-window analysis.[5] In panel (a) we see that in the 24-12 months preceding a sudden stop, interest rates tend to be below their normal-times levels, which we think contributes to the buildup of leverage and imbalances in the economy. Then, in the 12 months immediately preceding the sudden stop, interest rates tend to increase sharply and remain significantly high for a long period of time. Panel (b) considers the case of the market sentiment factor and shows that the level of this variable also increases during sudden stops, implying that external factors matter for such episodes. Even though the market sentiment factor increases considerably in the months preceding a sudden stop, this increase is not as pronounced as for the interest rate.

spend, on average, 9.2 percent of the time in sudden stop states at a monthly frequency, but only around 2.4 percent of the time if measured at an annual frequency.

[5]Appendix A (available online) shows the corresponding analysis for the output gap. Our results are in line with existing evidence (Korinek and Mendoza, 2013): The output gap is 1 percent above normal times in the months preceding a sudden stop and 1.5 percent below normal times afterwards.

Panel (c) shows the evolution of interest rate volatility around sudden stops. Here we show the volatility of the overall interest rate (*i.e.*, risk-free rate *plus* country spread). Interest rate volatility is typically below the normal-times mean in the 24-12 months preceding the sudden stop, perhaps also contributing to the buildup of imbalances in the economy. Then, volatility rises slowly in the months preceding the sudden stop until it peaks around the sixth month after the event begins and then gradually returns to its normal-times level.

Panel (d) shows the volatility of market sentiment around sudden stops. By construction, this component excludes changes in the purely idiosyncratic component of the country spread and could, in principle, be considered exogenous for any individual country. The figure shows that sudden stops are typically preceded by a very sharp spike in market sentiment volatility, around six months before the sudden stop. Then, after a few months of calm, market sentiment volatility tends to spike up again before converging back to normal levels.

The evolution of the level and volatility of the market sentiment factor shown in panels (b) and (d) of Figure 3, together with the fact that this factor accounts for the majority of the variation in EMEs' interest rates, implies that, around sudden stops, EMEs are subject to risks associated with interest rate movements that are exogenous to countries' fundamentals. The model we propose in the next section assumes such exogeneity of interest rates.

2.4 Regime-Switching VAR Model of Interest Rates and Output

We now estimate a regime-switching VAR model to capture the time-varying nature of interest rate volatility in our sample of EMEs. We will then use the estimated VAR to solve and simulate the theoretical model introduced in the following section.

We assume that the dynamics of the output gap (z_t) and the interest rate (r_t) across EMEs can be characterized by a single regime-switching VAR model. As is common in the literature, we allow for contemporaneous correlation and dynamic feedback between the output and interest rate processes. The random vector $(z_t, r_t)'$ has the following VAR specification:

$$\begin{pmatrix} z_t \\ r_t \end{pmatrix} = A_0 + A_1 \begin{pmatrix} z_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^r \end{pmatrix}. \tag{1}$$

The draws of the shock vector $(\epsilon_t^z, \epsilon_t^r)'$ are independent across time, and they have a Gaussian distribution with zero mean and a covariance matrix that itself has a stochastic evolution:

$$\Sigma_t = \begin{pmatrix} (\sigma^z)^2 & \rho \cdot \sigma^z \cdot \sigma_t^r \\ \rho \cdot \sigma^z \cdot \sigma_t^r & (\sigma_t^r)^2 \end{pmatrix}. \tag{2}$$

We assume that the volatility of the external interest rates takes on two values, $\sigma_t^r \in \{\sigma_L^r, \sigma_H^r\}$, with $\sigma_H^r > \sigma_L^r > 0$. The switching between these regimes is governed by a first-order Markov process with transition matrix Π.

We estimate the parameters that rule the regime-switching VAR given by (1) for our sample of EMEs by maximum likelihood. We assume that the (z_t, r_t) time series for each country are an independent realization of a single data-generating process characterized by (1). Hence, we maximize a single joint likelihood function that integrates every country's data at once. We use annual data for our estimation because it is the appropriate frequency for the calibration of our theoretical model in Sections 3 and 4. Quarterly GDP figures were annualized and then log-linearly detrended, and monthly interest rate data were averaged arithmetically.

The estimated VAR process is

$$\begin{pmatrix} z_t \\ r_t \end{pmatrix} = \begin{pmatrix} 0.0069 \\ 0.0020 \end{pmatrix} + \begin{pmatrix} 0.7093 & -0.0936 \\ 0.0467 & 0.8653 \end{pmatrix} \begin{pmatrix} z_{t-1} \\ r_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^z \\ \epsilon_t^r \end{pmatrix}, \tag{3}$$

and the covariance and transition matrices are composed of:

$$\begin{aligned} &\sigma^z = 0.0426, \quad \rho = -0.5228, \quad \pi_L = 0.9565, \\ &\sigma_L^r = 0.0094, \quad \sigma_H^r = 0.0833, \quad \pi_H = 0.8238. \end{aligned}$$

The ergodic mean of the output and interest rate processes can be obtained by inverting the VAR as follows:

$$E \begin{pmatrix} z_t \\ r_t \end{pmatrix} = (I - \hat{A}_1)^{-1} \hat{A}_0 = \begin{pmatrix} 0.0171 \\ 0.0208 \end{pmatrix},$$

where $\hat{A}_0$ and $\hat{A}_1$ denote the estimated matrices in (3). The long-run average of the external interest rate is thus 2.08 percent.

The two regimes of the VAR have considerably different interest rate volatilities. In the low-volatility regime, the standard deviation of interest rate shocks is small ($\sigma_L^r = 0.0094$), leading to a low refinancing risk for bond holdings. In contrast, in the high-volatility regime, the standard deviation is 9 times greater ($\sigma_H^r = 0.0833$), inducing higher uncertainty about future access to debt financing for the economy. Our estimation implies that that the long-run probability of being in the high-volatility state is 20 percent. We rely on our estimation to simulate the theoretical model that we present next.

3 Model

We consider a framework in the vein of the models proposed by Jeanne and Korinek (2018) and Bianchi and Mendoza (2018) to study macroprudential policies in small open economies subject to sudden stops driven by asset price collapses. We introduce external shocks to the level and volatility of interest rates into this framework.

3.1 Environment

Consider a small open economy inhabited by a continuum of identical atomistic households indexed by $i \in [0, 1]$. Households have preferences over sequences of consumption, $c_{i,t}$, given by

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \tag{4}$$

where period utility exhibits constant relative risk aversion, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, and $\beta < 1$.

Households receive two types of income in every period: the payoff of an asset, which we model as a Lucas tree, and an endowment. The evolution of both types of income is described by the same stochastic process, which we denote by d_t. In particular, in a given period t, the asset pays dividend αd_t, while households also receive $(1 - \alpha) d_t$ units of consumption as an endowment, where $\alpha \in [0, 1]$. Asset shares are only traded domestically.

Households have access to international financial markets by issuing one-period bonds with an exogenous gross return $1 + r_t$. The external interest rate follows a stochastic process, but debt contracts are locally risk free: A household knows at time t the interest rate that it must pay next period for its outstanding bonds, but it does not know the interest rate that it will face next period if it decides to refinance its stock of debt.

Note that, while consumption goods derived from the asset dividends can be traded internationally, the stocks of the tree can only be held by domestic households.

Household i's budget constraint in period t is then given by

$$c_{i,t} + q_t s_{i,t+1} + \frac{b_{i,t+1}}{1 + r_t} = (1 - \alpha) d_t + (q_t + \alpha d_t) s_{i,t} + b_{i,t}, \tag{5}$$

where we have normalized the price of consumption to unity. In equation (5), $b_{i,t}$ denotes the face value of bonds and $s_{i,t}$ the asset holdings of household i at the beginning of period t. Hence, household i's income in t is given by the right-hand side of (5), where q_t denotes the price at which assets are traded in t. Households can use their income to consume, $c_{i,t}$; exchange assets, $s_{i,t+1}$; or change their net foreign asset position by buying or selling bonds, $b_{i,t+1}$. We follow the convention that a positive b_t represents savings by the households overseas, whereas negative positions represent external household debt.

The key friction in this economy arises from a collateral constraint. In particular, a household's borrowing at time t is constrained by a fraction κ of the value of its domestic asset holdings:

$$-\frac{b_{i,t+1}}{1+r_t} \leq \kappa q_t s_{i,t+1}, \tag{6}$$

where κ determines how stringent the constraint is. Hence, households' access to international financial markets in order to smooth consumption and fund their stock purchases is limited.

We extend this model to allow for shocks to the level and volatility of international interest rates. In particular, we assume that shocks to income z_t—where income is given by $d_t = d\exp(z_t)$—and interest rates r_t evolve according to the stochastic process described by the regime-switching VAR process in (1) and (2), which we estimated in Section 2.

3.2 Household's Problem

In order to highlight the pecuniary externalities present in a competitive equilibrium of our economy, we rewrite the problem of the representative household in recursive form. The aggregate states in the household's problem are the aggregate level of savings, B, and the current realization of the stochastic shocks, which we denote by $X \equiv (z, r, \sigma^r)$. The individual states of a household are its holdings of bonds, b, and stocks of the tree, s. We denote by $V(b, s, B, X)$ the value of a household's problem with portfolio (b, s) when the aggregate states are B and X. Households take as given a perceived law of motion for aggregate bonds, $B' = \mathcal{B}(B, X)$, to form expectations of future prices. Then, the Bellman equation of the problem is

$$V(b, s, B, X) = \max_{c, b', s'} u(c) + \beta \mathbb{E}[V(b', s', B', X')|X] \tag{7}$$

subject to

$$\begin{aligned} c + \mathcal{Q}(B, X)s' + \frac{b'}{1+r} &= (1-\alpha)\, d(X) + [\mathcal{Q}(B, X) + \alpha d(X)]s + b, \\ -\frac{b'}{1+r} &\leq \kappa \mathcal{Q}(B, X)s', \\ B' &= \mathcal{B}(B, X). \end{aligned}$$

In the previous expression, $\mathcal{Q}(B, X)$ is the market value of the tree. This price is determined in equilibrium and depends on the aggregate state of the economy. As we will make clear in the next subsection, a recursive competitive equilibrium requires that $\mathcal{B}$ is consistent with optimal individual decision rules and that $\mathcal{Q}$ ensures the clearing of the market for stocks of the tree.

The solution to the household's problem satisfies the following Euler equation for bonds

$$u'(c(b,s,B,X)) - \mu(b,s,B,X) = (1+r)\beta\mathbb{E}\{u'(c(b',s',\mathcal{B}(B,X),X'))|X\}), \tag{8}$$

and for stocks of the tree:

$$\mathcal{Q}(B,X) = \frac{\beta\mathbb{E}\{u'(c(b',s',\mathcal{B}(B,X),X'))[\mathcal{Q}(\mathcal{B}(B,X),X')+\alpha d(X')]|X\}}{u'(c(b,s,B,X))\left(1-\frac{\kappa\mu(b,s,B,X)}{u'(c(b,s,B,X))}\right)}, \tag{9}$$

where $\mu \geq 0$ is the multiplier on the borrowing constraint.

3.3 Competitive Equilibrium

A competitive equilibrium is defined by sequences of allocations $\{c_{i,t}, b_{i,t+1}, s_{i,t+1}\}_{t=0}^{\infty}$ for every household i and of asset prices $\{q_t\}_{t=0}^{\infty}$ such that households solve their optimization problems and the asset market clears. Given that all the households are identical and that they only face aggregate shocks, asset market clearing implies that $s_t = 1$ in every period. The following is the formal definition of a recursive competitive equilibrium of this economy.

Definition 1 *A recursive competitive equilibrium of this economy consists of a pricing function, $\hat{\mathcal{Q}}(B,X)$; a perceived law of motion for aggregate bond holdings, $\hat{\mathcal{B}}(B,X)$; and decision rules for households, $\hat{b}(b,s,B,X)$ and $\hat{s}(b,s,B,X)$, with associated value function $\hat{V}(b,s,B,X)$ such that:*

1. *Given $\hat{\mathcal{Q}}(B,X)$ and $\mathcal{B}(B,X)$, households' decision rules, $\hat{b}(b,s,B,X)$ and $\hat{s}(b,s,B,X)$, and the associated value function, $\hat{V}(b,s,B,X)$, solve the recursive problem of the household given by (7).*

2. *$\hat{\mathcal{B}}(B,X)$ is consistent with the actual law of motion for bond holdings; $\hat{\mathcal{B}}(B,X) = \hat{b}(B,1,B,X)$.*

3. *Asset markets clear. In particular, $\hat{\mathcal{Q}}(B,X)$ is such that $\hat{s}(B,1,B,X) = 1$.*

Given the definition of the equilibrium, note that the equilibrium level of bonds can be characterized by a simple function of the aggregate state variables, $B' = \hat{\mathcal{B}}(B,X)$, which, together with the resource constraint, defines consumption as a function of aggregate state variables, $\hat{\mathcal{C}}(B,X)$.

In our framework, a sudden stop in external financing arises endogenously as a consequence of the households' borrowing decisions. For high levels of leverage, if the borrowing constraint binds, households are forced to sharply reduce their debt, which is only possible through a drastic decline in consumption. This drop causes a sharp decline in asset prices, generating a pecuniary externality because of the fact that households do not internalize the effect of their decisions on prices. In turn, the value of collateral is reduced, which further tightens the borrowing constraint and induces more deleveraging. The feedback between asset price reductions, deleveraging, and drops in consumption generates a sudden reversal of the capital flows into the country.

When the external borrowing rate is lower than the households' discount factor, the households face a fundamental trade-off between impatience and insurance (Korinek and Mendoza, 2013). They have an incentive to borrow in order to consume in advance. Nonetheless, for high levels of borrowing, a crisis is more likely to happen, and households have the incentive to save and avoid the crisis region. In the next section, we illustrate numerically the interaction between these two motives.

How do shocks to external risks affect households' decisions? Even though we will discuss in detail the effects of external shocks on the competitive equilibrium of the economy in Section 4, we provide a first look at these effects in what follows.

Changes in the level of external interest rates affect the marginal cost of borrowing, as shown in the right-hand side of equation (8). Low interest rates imply low marginal costs of borrowing that incentivize households to acquire more debt and increase consumption in the current period, c_t. Concurrently, changes in the interest rate have implications for asset prices through their effect on the stochastic discount factor, as can be seen in equation (9), and their effect on how future dividends are discounted by households. Everything else constant, low interest rates increase current asset prices, q_t, because the present value of future dividends increases. Hence, shocks to external interest rates lead to more volatile capital flows and domestic asset prices. Note that the persistence of these shocks will also matter for households' optimal decisions.

Changes in the volatility of interest rates, while keeping their levels constant, affect households' optimal decisions primarily by increasing the volatility of future consumption. An increase in volatility implies that debt becomes a worse instrument for hedging against future income shocks because it increases the risk of refinancing debt in the future. If future consumption becomes more volatile, but its expected level remains relatively unchanged, then $\mathbb{E}_t u'(c_{t+1})$ increases (by Jensen's inequality), which in turn implies that current consumption must decline for (8) to hold for fixed r_t. Therefore, households reduce their debt, which leads to a reduction in current consumption, c_t. However, changes in volatility also have implications for expected future consumption, depending on a country's net foreign asset (NFA) position at the time of the interest rate shock. Overall, and similarly to shocks to the level of external interest rates, shocks to their volatility lead to more volatile capital flows and asset prices. In the rest of the paper we explore in detail these mechanisms and how they shape optimal policy. However, first we consider the problem of a social planner that internalizes the effect of external borrowing on the value of collateral.

3.4 Social Planner's Problem

We consider a social planner that can only choose the level of aggregate debt, subject to the economy's borrowing constraint. The planner cannot directly intervene in the domestic asset markets where trading takes place between households, so it tries to affect the equilibrium value of collateral indirectly by altering the economy's borrowing decisions. We assume that the planner cannot commit to future policies, and we solve for the constrained efficient allocation implemented through time-consistent policies.[6]

We follow Klein et al. (2005) and restrict attention to planner's policy rules that depend only on the current state of the economy. This restriction implies that the policy rule of the planner is given by a simple function of the current states, (B, X), that maps them into levels of aggregate bonds, $B' = \Psi(B, X)$. In this case, we show in Appendix B (available online) that the planner's problem can be stated as follows. Given an arbitrary future policy rule, $\Psi(B, X)$, and the associated asset pricing function, $\mathcal{Q}(B, X)$, the social planner chooses c and B' that solve the following Bellman equation:

$$W(B, X) = \max_{c, B'} \{u(c) + \beta \mathbb{E}[W(B', X') | X]\} \tag{10}$$

subject to

$$c + \frac{B'}{1+r} = d(X) + B, \tag{11}$$

$$-\frac{B'}{1+r} \leq \kappa \bar{\mathcal{Q}}(B, B', X), \tag{12}$$

and the valuation of collateral is consistent with the household's trading of the stocks of the

[6]We follow Jeanne and Korinek (2018) and Bianchi and Mendoza (2018) in setting up the problem of the constrained social planner. Benigno et al. (2013) point out that the solution to the constrained planner's problem differs from the Ramsey solution. This difference occurs because the former planner is restricted to a particular set of policy instruments.

tree:

$$\bar{\mathcal{Q}}\left(B,B',X\right)=\beta\mathbb{E}\left[\left.\frac{u'\left(d(X')+B'-\frac{\Psi(B',X')}{1+r'}\right)\left[\mathcal{Q}(B',X')+\alpha d(X')\right]}{u'\left(d(X)+B-\frac{B'}{1+r}\right)}\right|X\right]. \tag{13}$$

The planner's decision now internalizes the fact that increasing households' savings affects equilibrium asset prices, which in turn alters the value of collateral in the borrowing constraint.

The literature has shown that this type of model is prone to multiple equilibria. In order to focus on the effects of interest rate risks, sudden stops, and macroprudential policy, we make the following technical assumption.[7]

Assumption The parameters and stochastic processes of the economy are such that the equilibrium pricing function, $\bar{\mathcal{Q}}\left(B,B',X\right)$, satisfies $1+\kappa\left(1+r\right)\xi(B,X)>0$, where

$$\xi(B,X) \quad = \quad \frac{\partial\bar{\mathcal{Q}}(B,\Psi(B,X),X)}{\partial B'}. \tag{14}$$

Under the previous assumption, we can derive the following lemma to characterize the social planner's optimal allocations.

Lemma 3.1 *The functions that solve the planner's problem, $c=\mathcal{C}(B,X)$ and $B'=\hat{\Psi}(B,X)$, must satisfy the following condition:*

$$u'(\mathcal{C}(B,X))-\mu(B,X)=\left(1+r\right)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X'))+\kappa\mu(B',X')\psi(B',X')\right] \tag{15}$$

where $\mu(B,X)$ is the Lagrange multiplier on the collateral constraint (12),

$$\psi(B,X) \quad = \quad \frac{\partial\bar{\mathcal{Q}}(B,\hat{\Psi}(B,X),X)}{\partial B},\ \ \textit{and} \tag{16}$$

$$\mathcal{C}(B,X) \quad = \quad B+d(X)-\frac{\hat{\Psi}(B,X)}{1+r}. \tag{17}$$

Proof See Appendix B.1 (available online).

To gain some intuition on how the planner internalizes the pecuniary externality, consider the case in which the collateral constraint does not bind in the current period, $\mu(B,X)=0$. In this case, equation (15) becomes

$$u'(\mathcal{C}(B,X))=\left(1+r\right)\beta\mathbb{E}\left[u'(\mathcal{C}(B',X'))-\kappa\mu(B',X')\psi(B',X')\right]. \tag{18}$$

[7]See Jeanne and Korinek (2018) and Korinek and Mendoza (2013) for more details on this assumption.

The planner's intervention considers not only the possibility of a binding borrowing constraint and how tight it is through the $\mu(B', X')$ term, which formally defines the *severity* of a crisis, but also the risk associated with the size of the price externality through the $\psi(B', X')$ term, which formally defines the *ability of the planner* to shape a crisis. It can be shown that when the constraint is non-binding, then (see Appendix B.1 online)

$$\psi(B, X) = -\frac{u''(\mathcal{C}(B, X))}{u'(\mathcal{C}(B, X))}\mathcal{Q}(B, X), \tag{19}$$

which implies that the ability of the planner depends on the level of asset prices and the coefficient of absolute risk aversion of the representative household.

In the problem of the planner, we assumed that an arbitrary future policy rule, $\Psi(B, X)$, and the implied asset pricing function, $\mathcal{Q}(B, X)$, are taken as given. Hence, the current planner can only affect the pricing function by choosing B' and then having the future planner make decisions based on $\Psi(B', X')$, rather than by committing to B' and B''. In Section B of the online appendix, we provide an expression for $\xi(B, X)$ that shows explicitly how it relates to having the planner take future policy rules as given.

Given the characteristics of the social planner's problem, it is straightforward to define a recursive constrained efficient allocation, conditional on arbitrary future planners' policy rules. Our definition of a constrained efficient allocation further requires that these policy rules be time consistent. In other words, we require that the policy that solves the strategic game being played by sequential planners is a fixed point, deriving in a Markov stationary policy rule. We provide further details and formal definitions of these concepts in the online appendix.

Shocks to the first and second moments of the world interest rate have important implications for the pecuniary externality. For instance, lower interest rates exacerbate "overborrowing" in the competitive equilibrium because borrowing becomes cheaper, which, in turn, increases the *severity* of crises, but also the *ability of the planner* to dampen the crises. Lower volatility of interest rates also amplifies the problem of overborrowing by incentivizing households to acquire more debt. The planner internalizes how these shocks affect asset prices and the value of collateral, as shown in equation (15). The rest of the paper focuses on how shocks to the level and volatility of interest rates affect the planner's decisions through the incidence and severity of crises. However, first we analyze the decentralization of the social planner's allocations by means of a particular instrument, a tax on debt.

Decentralization To explore how the planner responds to the external shocks faced by the economy, we will focus on analyzing the state-contingent macroprudential tax on debt that decentralizes the constrained efficient allocation.[8] Comparing the equations that characterize the solution of the competitive equilibrium and the planner's problem (equations (8) and (15), respectively), it can immediately be seen that the wedge on households' gross interest rate that implements the allocation of the planner's problem is

$$\tau(B, X) = \frac{\mathbb{E}[\kappa\psi(B', X')\mu(B', X')|X]}{\mathbb{E}[u'(c(B', X'))|X]}, \tag{20}$$

where $B' = \hat{\Psi}(B, X)$ is the optimal level of savings chosen by the planner when initial savings are B, and shocks X are realized. From equation (20) we see that the size of the planner's intervention is determined by the expected marginal welfare gain of reducing households' indebtedness: The value of reducing households' debt by a unit is equal to the increase in the value of collateral, $\kappa\psi(B', X')$, times the marginal value of relaxing the collateral constraint, $\mu(B', X')$. In Section 4.4, once we have calibrated the model, we return to a detailed analysis of the forces shaping the optimal tax in response to external risks.

4 Quantitative Analysis

The results derived in the previous section show that the level and volatility of external interest rates partially shape the pecuniary externality present in the model. This feature implies that, qualitatively, the optimal tax on debt given in (20) should respond to changes in these variables. However, the relevance of interest rate levels and their volatility in shaping macroprudential policy is ultimately a quantitative question. In this section we choose parameter values for the model in order to use it as a quantitative tool to address the previous matter.

4.1 Parameterization and Numerical Solution

To choose parameters, we either consider values in the existing literature or use data to map to empirical counterparts of the model. As previously stated, we consider a utility function with constant relative risk aversion, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

Table 2 presents the baseline parameterization of the model for an annual frequency. The preference parameters are standard in the literature on small open economies. Our choice of the relative risk aversion parameter value, $\gamma = 2$, is at the lower end of the values used in the literature on emerging market business cycles. Hence, the quantitative effects of volatility on

[8] There are other policy instruments that can decentralize the constrained efficient allocation. Bianchi and Mendoza (2018) provide some examples of such instruments. They focus on a tax on debt, as we do here because we believe it provides a very intuitive perspective on how to influence capital flows across countries.

real allocations and asset prices that we show are, in principle, conservative. We calibrate the discount factor, β, to a standard value such that the discount rate, $\beta^{-1} - 1 \approx 0.04$, is well above the long-run interest rate of 2.1 percent estimated in Section 2.

To calibrate the parameter in the collateral constraint, κ, as well as the share of income derived from the asset, α, we rely on model simulations. In particular, we choose these two parameters by trying to match two features of the data: (i) the annual increase in the level of interest rates observed in our data during sudden stops and documented in Section 2, and (ii) the unconditional probability of a sudden stop at an annual frequency of 2.4 percent obtained from our data. This probability is somewhat below the 3 percent that is usually used in the literature as a benchmark (Korinek and Mendoza, 2013). We calibrate $\kappa = 0.1$, more than twice the value considered by Jeanne and Korinek (2018) of 0.04, and $\alpha = 0.25$, which is much closer to the value of 0.2 calibrated by Jeanne and Korinek (2018), by considering the decline in debt and the value of assets of small and medium enterprises in the United States between the second quarter of 2008 and the second quarter of 2009.

For our estimated parameters, we normalize the mean of the dividends process, d, to easily interpret the measurements of consumption, savings, and asset prices relative to the mean annual income. Lastly, we rely on the estimation carried out in Section 2 to discipline the stochastic process followed by z, r, and σ_r.

Table 2: Baseline parameterization

Parameters Set Independently			
		Value	Target
Time discount	β	0.96	Standard value
Relative risk aversion	γ	2	Standard value
Parameters Set by Simulation			
		Value	Target
Share of income from asset	α	0.25	Interest rate increase in sudden stops
Collateral constraint	κ	0.1	Probability of sudden stops
Parameters Estimated with Annual Data			
		Value	Target
Average income	d	1	Normalization
Average interest rate	r	2%	Estimation in Section 2
Average volatility of interest rates	σ	2.4%	Estimation in Section 2

We use a global solution method to characterize the recursive competitive equilibrium of the economy in a discretized version of the aggregate state space. Appendix C (available online) describes in detail our algorithm. The discretization of our stochastic process deserves additional comments. We follow Terry and Knotek II (2011) in discretizing the estimated VAR process and use a two-dimensional variation of the Tauchen (1986) method that allows for different levels of variance of the shocks. We use a grid of 7 points for output shocks and 15 points for the interest rate to better capture the effects of changing volatility of the latter variable. We truncate the grids in order to include 95 percent of the probability mass of shocks at the ergodic distribution for the high-volatility regime, which was approximated by simulating the VAR for 1 million periods. The next section describes our numerical solution.

4.2 Competitive Equilibrium and Planner's Policy Functions

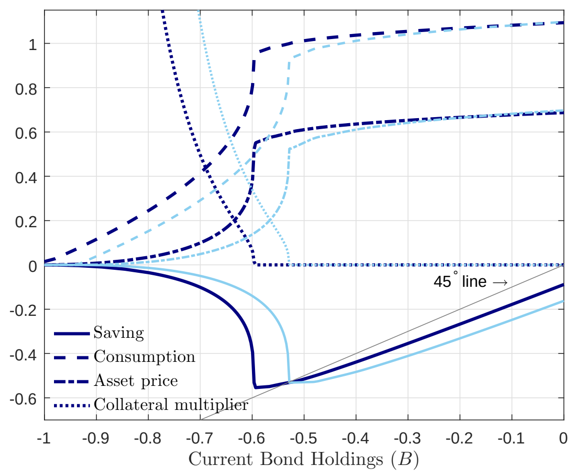
We start by analyzing the competitive equilibrium and the social planner's policy functions as well as the response of these functions to the exogenous shocks in the model—in particular, shocks to the level and volatility of interest rates.

Figure 4 plots the policy functions for the competitive equilibrium of this economy as a function of the endogenous state, B. In particular, the dark blue lines plot these functions for the average income and interest rate levels and a low volatility of interest rates. The savings rule, $\hat{\mathcal{B}}(B, X)$ (solid line), reflects the incentive of households to borrow (the rule lies below the 45 degree line), driving the economy nearer to the sudden-stop or crisis region—defined by the levels of bond holdings for which the borrowing constraint is binding and the collateral multiplier (dotted line) is positive ($\hat{\mu}\,(B, X) > 0$)—close to which the rule becomes downward sloping, reflecting the need to deleverage in times of crisis. In this situation, households must reduce their consumption in order to lower their stock of debt, as displayed by the consumption policy function, $\hat{\mathcal{C}}(B, X)$ (dashed line), leading to a consequent drop in asset prices, $\hat{\mathcal{Q}}(B, X)$ (dash-dotted line). The drop in the value of collateral forces a deleveraging that feeds back into further consumption cuts and asset price falls, ad infinitum.

Let us consider a negative income shock. The light blue lines in Figure 4 show the policy functions for the lower level of income. The negative shock leads to a decline in consumption and asset prices, as well as a decrease in savings. Note that these changes are much stronger closer to the crisis region and that the shock can easily move the economy from the non-crisis to the crisis region.

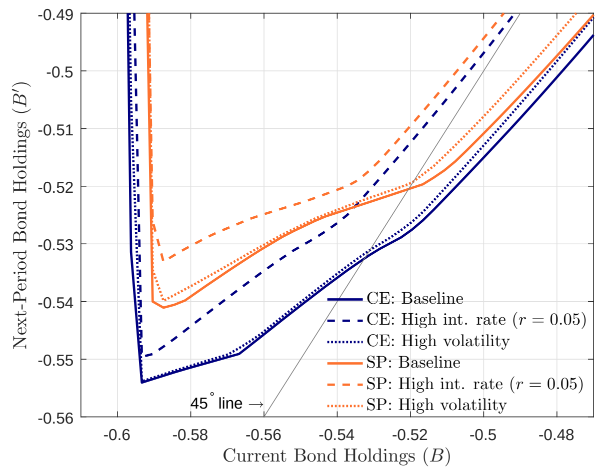
How does the social planner's (SP) saving policy ($\hat{\Psi}(B, X)$) compare with the one in the competitive equilibrium (CE)? Figure 5 compares the savings rules for households in the CE (solid blue line) and the solution to the planner's problem (solid orange line) for the average exogenous state that we refer to as baseline. As the level of current bond holdings approaches the crisis region, these policy functions become more different, as the planner increases savings relative to the agents in the CE in order to avoid a sudden stop. Hence, the planner's policy function is shifted upward relative to the one in the CE. The figure also plots these functions for a higher level of interest rates (dashed lines) and for the high level of volatility of interest

Figure 4: Policy rules in the CE



Notes: Dark blue lines represent policy functions for a given exogenous state of the economy: $d = 1, r = 0.02$, and $\sigma^r = \sigma^r_L = 0.01$ (average interest rate and low volatility). Light blue lines depict the same functions for a lower level of income, $d = 0.94$, keeping $r = 0.02$ and $\sigma^r = 0.01$.

Figure 5: Savings policy rules: CE vs. SP



Notes: Lines depict savings policy rules for the CE (blue lines) and the SP (orange lines) given average income ($d = 1$). Solid lines consider baseline state $r = 0.02, \sigma^r = 0.01$. Rules for high int. rates ($r = 0.05, \sigma^r = 0.01$) are depicted by dashed lines and high volatility ($r = 0.02, \sigma^r = 0.08$) by dotted lines.

rates (dotted lines). In line with the effects described in Section 3, higher interest rates lead to an increase in savings for both the CE and the SP. However, note that the policy function of the agents in the CE is more sensitive to changes in the interest rate than the SP's policy function. This difference reflects the fact that the planner tries to avoid overborrowing for low levels of interest rates. Turning to the effects of an increase in interest rate volatility, note that higher volatility also generates an increase in savings, particularly for current levels of bond holdings far from the crisis region. This result is in line with precautionary motives given increases in the volatility of future consumption. However, note that for the particular exogenous state considered, shifts in saving policies for given changes in the volatility of interest rates are considerably smaller than for changes in their levels.

The effects of volatility shocks shown in Figure 5 do not provide a full picture of changes in behavior by private agents and the SP. Figure 6 considers a higher interest rate level in the current period and plots the CE's and SP's savings rules for different levels of volatility. As the figure shows, for a high current level of interest rates, the effect of higher volatility of interest rates on savings is reversed and leads to an increase in borrowing for both the CE and the SP. To explain the difference in behavior between Figures 5 and 6, it is important to understand the difference between a mean preserving shock to income and one to interest rates. For a small open economy standing in period t, aggregate consumption in $t + 1$ is determined by its income and net export (nx_{t+1}) that same period:

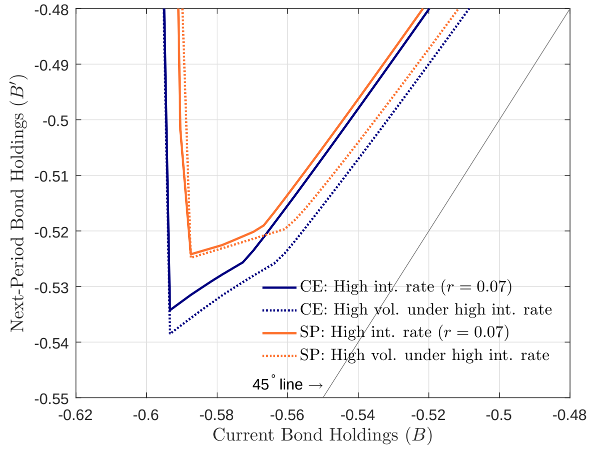
$$c_{t+1} = d_{t+1} - \underbrace{\left(\frac{1}{1+r_{t+1}} (b_{t+2} - b_{t+1}) - \left(\frac{r_{t+1}}{1+r_{t+1}} \right) b_{t+1} \right)}_{\equiv nx_{t+1}}, \tag{21}$$

where we have further decomposed net exports into a term that captures changes in bond holdings and another that captures a country's NFA position. For greater *income* volatility, the economy can self-insure by relying on its bond holdings and adjusting its net exports—and therefore NFA position—without having any implications for the effects of the actual realization of the income shock on consumption. However, the effects of the realization of an *interest rate shock* depend directly on the values of b_{t+1} and b_{t+2}. We argue that the first term in nx_{t+1} reflects mainly roll-over risk, while the second one reflects the country's exposure to its NFA position. Consider a debtor country facing average or low levels of r_t. Then, the effect of a mean-preserving spread on interest rates is similar to that of one on income ($\frac{d}{dr}\frac{r}{1+r}$ is close to one for r close to zero), and the country saves to reduce both roll-over risk and volatile wealth effects in light of greater volatility of interest rates. However, if the country faces high interest rates, negative wealth effects generated by interest rate shocks are dampened as two things happen: (i) the agents save more endogenously and (ii) the probability of being in the flatter part of $\frac{r_{t+1}}{1+r_{t+1}}$ increases (this function of r_{t+1} is flatter for high values of r_{t+1}). In addition, the first term in nx_{t+1} becomes less effective at hedging risks if r remains high and, most importantly, the effect of an increase in interest rates on a negative NFA is less harmful than a decrease. Hence, decreasing savings becomes optimal. In online Appendix D we provide a simple three-period version of the model to show how decreasing savings is the optimal result of the forces just described rather than a numerical result of our model.

Figure 7 plots the optimal macroprudential tax function in (20) for the exogenous states considered in Figures 5 and 6. The figure shows how the asymmetric effects of volatility shocks for different levels of the interest rate can translate into opposite shifts in the optimal tax function. Furthermore, differences in the tax function for different levels of volatility at average interest rates show that the configuration of this policy can be quite complex. Therefore, in Section 4.4 we return to a careful description of these functions and provide a decomposition of the forces that shape its complexity.

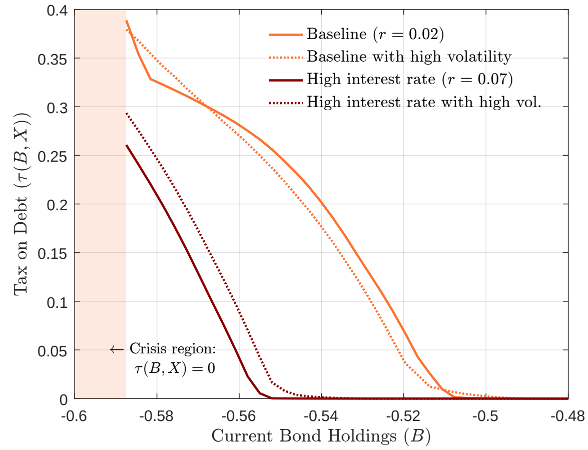
Our previous results provide a limited picture of how macroprudential policy should respond to volatility shocks. The optimal tax is a complex function of the economy's entire state space, and not all state levels are equally relevant because some of them seldom occur. Therefore, in the following section we rely on simulations of the model to explore, first, its predictions in

Figure 6: Effects of volatility shocks on savings rules under high interest rates



Notes: Solid lines depict savings policy rules for the CE (blue lines) and the SP (orange lines) given average income ($d = 1$), a high interest rate ($r = 0.07$), and low volatility ($\sigma^r = 0.01$). Dashed lines consider an increase in volatility to $\sigma^r = 0.08$ while keeping income and the level of interest rates unchanged.

Figure 7: Response of optimal macroprudential tax to changes in volatility



Notes: Lines depict the optimal tax function for average income, $d = 1$, and a given level and volatility of int. rates. The orange solid (dashed) lines correspond to the case of low (high) volatility and average int. rate ($r = 0.02, \sigma_r = 0.01$). Dark red lines consider each case for high int. rates ($r = 0.02$).

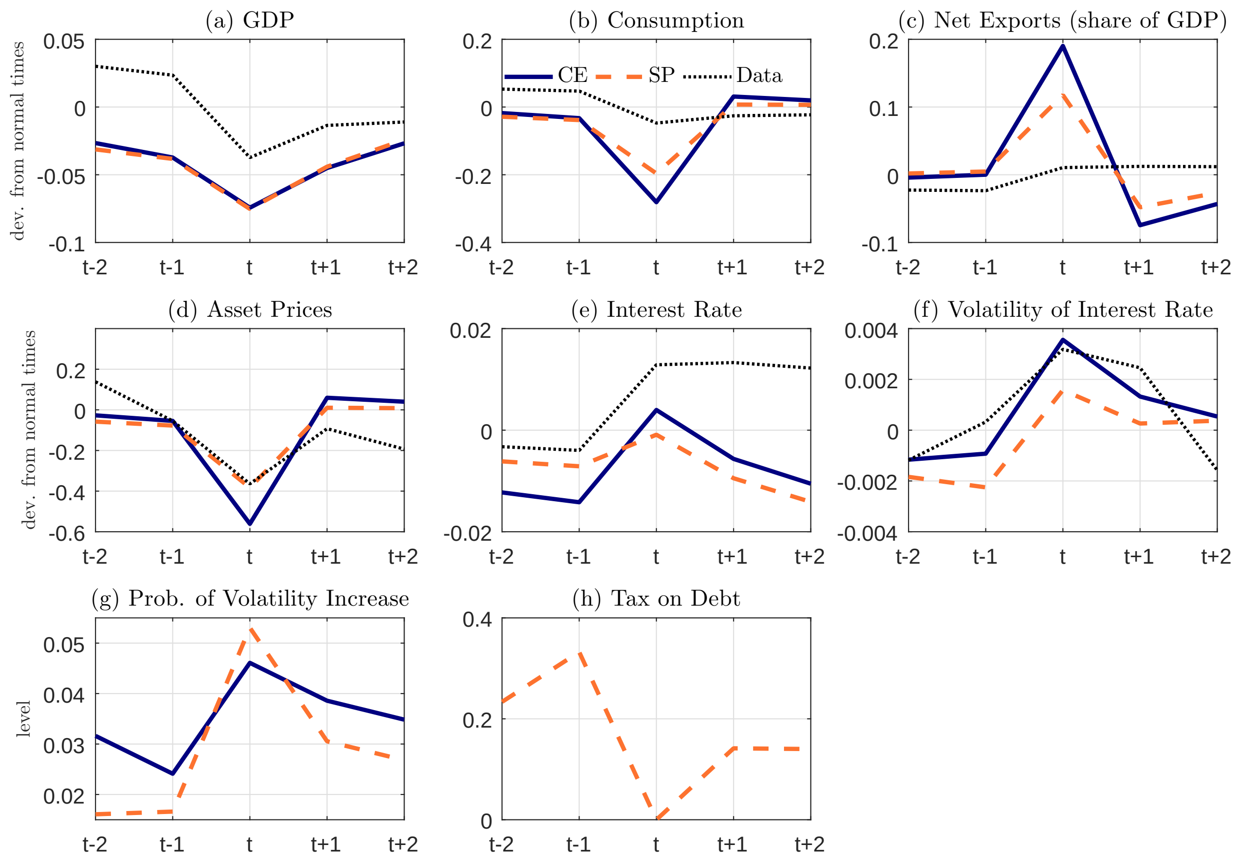
generating sudden stops and, second, the response of macroprudential policy to volatility shocks.

4.3 Sudden Stop Dynamics and Long-Run Moments

We now turn to the dynamics of the economy around sudden stop episodes. We simulate the model, with and without imposing the optimal macroprudential tax (20) that decentralizes the social planner's allocations, for 100,000 periods and define a sudden stop as a period in which the collateral constraint binds. Panel (b) of Table 3 reports some long-run moments of our simulation. The probability of a sudden stop without taxes is 1.9 percent, which is consistent with the fact that these are rare events that occur 2.4 percent of the time in our data at an annual frequency. While the model does a good job of matching the prevalence of sudden stops, it also generates a long-run negative net foreign asset (NFA) position of 49.5 percent of GDP, considerably higher than the 27.8 percent of the average for our sample of countries reported in Lane and Milesi-Ferretti (2007).

Regarding the dynamics of the model, Figure 8 depicts the competitive equilibrium's (solid blue lines labeled as CE) and social planner's (dashed red lines labeled as SP) variables around a typical sudden stop. In line with our empirical analysis, all variables unless otherwise noted are expressed relative to "normal times"—that is, periods in which the borrowing constraint is non-binding—and are plotted from $t-2$ to $t+2$, where t is the moment when the sudden stop occurs. In addition, and in order to contrast the model with the empirical evidence, we plot the evolution of these variables in the data (dotted black lines).

Figure 8: Event study of typical sudden stops



Notes: The measure of the volatility of interest rate shown in panel (f) is the absolute value of the change in the interest rate, $|r_t - r_{t-1}|$, which provides a proxy for annualized volatility in our data (dashed black line). Panels (a) through (f), with the exception of panel (c), present the data in percentage point deviations from normal-times levels (a value of $0.x$ is equivalent to an 'x' percent deviation). Panels (c), (g) and (f) consider raw levels and are not presented in terms of deviations of particular levels. As references, during normal times, net exports as a share of GDP are 0.01, the probability of an increase in volatility is 0.34, and the tax on debt is 0.09.

Panels (a) through (c) of Figure 8 show that the dynamics of output, consumption, and net exports are in line with the data. Table 3 complements Figure 8 by reporting that sudden stops in the model are characterized by large drops in output and consumption of 3.5 and 21.3 percent, respectively, as well as a significant increase of 19 percentage points in net exports as a share of GDP. While the model dynamics are in line with the data, the model generates much larger changes than observed in their empirical counterparts. Moreover, the model cannot match the fact that output is above average before sudden stops. However, given the simplicity of the model, we believe it does a remarkable job of replicating the behavior of all the variables considered around sudden stops.

Panels (d) through (f) of Figure 8 show the typical evolution of asset prices, interest rates,

Table 3: Sudden Stops and Long-Run Moments

(a) Sudden Stops			
	Comp. Equilibrium	Social Planner	Data
Change in GDP (%)	-3.5	-3.5	-6.0
Change in Consumption (%)	-21.3	-14.0	-9.4
Change in Net Exports/GDP (pp)	19.0	11.3	3.4
Change in Asset Prices (%)	-37.7	-24.4	-25.2
Change in Interest Rates (pp)	1.8	0.6	1.7
Change in Volatility (pp)	0.5	0.4	0.3
(b) Long-Run			
	Comp. Equilibrium	Social Planner	Data
Probability of Sudden Stop (%)	1.9	1.5	2.4
NFA position (share of GDP, %)	-49.8	-49.3	-27.8

Notes: Our estimated annual probability of a sudden stop ranges from 0.9 to 4.8 percent depending on the number of months with sudden stops that define such an event in a given year.

and the volatility of interest rates around sudden stops. Both in the model and in the data, sudden stops are associated with large drops in asset prices (37.7 and 25.2, respectively), but changes in the model are larger. The drops in asset prices occur as interest rates increase by 1.8 percentage points, which basically matches the interest rate increases of 1.7 percentage points in the data. The model is also consistent with the fact that sudden stops take place after periods of relatively low and stable interest rates (panels (e) and (f)) before the level and volatility spike at the time of the sudden stop. Sudden stop episodes in the model are also consistent with a pronounced increase in the probability of seeing an increase in the fundamental volatility of interest rates (unobserved in data). In line with measured volatility (panel (f)), panel (g) shows that the probability of an increase in σ_r is below its average of 0.34 prior to the sudden stop, before increasing by close to a factor of two at the time of the sudden stop.

We now turn to the social planner's allocations. Even though the probability of sudden stops only drops by 0.5 percentage point, the decline in consumption during sudden stops is considerably smaller than in the competitive equilibrium (14.0 versus 21.3 percent as shown in Table 3). These features imply that the optimal tax leads to welfare gains of 1.7 percent in terms of compensating variations in consumption. Figure 8 also shows that asset prices do not drop as much as in the competitive equilibrium and that even though the interest rate does not increase as much, the typical sudden stop occurs after periods of stable interest rates. On average, the optimal macroprudential tax is 9.3 percent. However, Figure 8 shows that, before a sudden stop, the tax is typically as high as 33.3 percent. Hence, optimal macroprudential taxes should increase before a sudden stop.

Our previous results imply that our simple calibrated model performs well in replicating the patterns observed in the data around sudden stops. Moreover, the differences in dynamics between the CE and SP allocations and prices are in line with the mechanisms described in previous sections. Hence, we also rely on our simulations to explore how macroprudential taxes should respond to volatility shocks. As shown in column (1) of Table 4, increasing macroprudential taxes in response to an increase in interest rate volatility in isolation is seldom optimal. A tax increase is optimal in only 13.1 percent of the cases. Moreover, half of these events call for no change in taxes at all. Even if we do not condition on constant income (column (2))—which would simplify policy implementation—increasing taxes is optimal only 25 percent of the time, and no change in taxes is optimal 40 percent of the time. Appendix E (available online) shows the distribution of optimal taxes conditional on volatility regimes and provides further evidence that higher taxes rarely prevail during times of high volatility.

Table 4: Response of Optimal Tax to Increase in Volatility of Interest Rates

Event	Mean-Preserving Spread in r_t only (1)	Mean-Preserving Spread in r_t (2)
Probability of event (%)	0.08	0.26
Prob. of tax increase conditional on event (%)	13.1	25.0
Prob. of tax unchanged conditional on event (%)	50.0	40.0
Prob. of tax decrease conditional on event (%)	36.9	35.0

Notes: The first type of event considers mean-preserving spreads in interest rates in isolation (income does not change), the second type includes changes in income. The average change in the optimal tax is -2.0 percent in both types of event.

4.4 Optimal Macroprudential Taxes

As Figure 7 shows, the optimal macroprudential tax is a complex function of the states of the economy. In this section we focus on these functions and provide a decomposition of the tax with the aim of better understanding the main forces shaping this policy instrument.

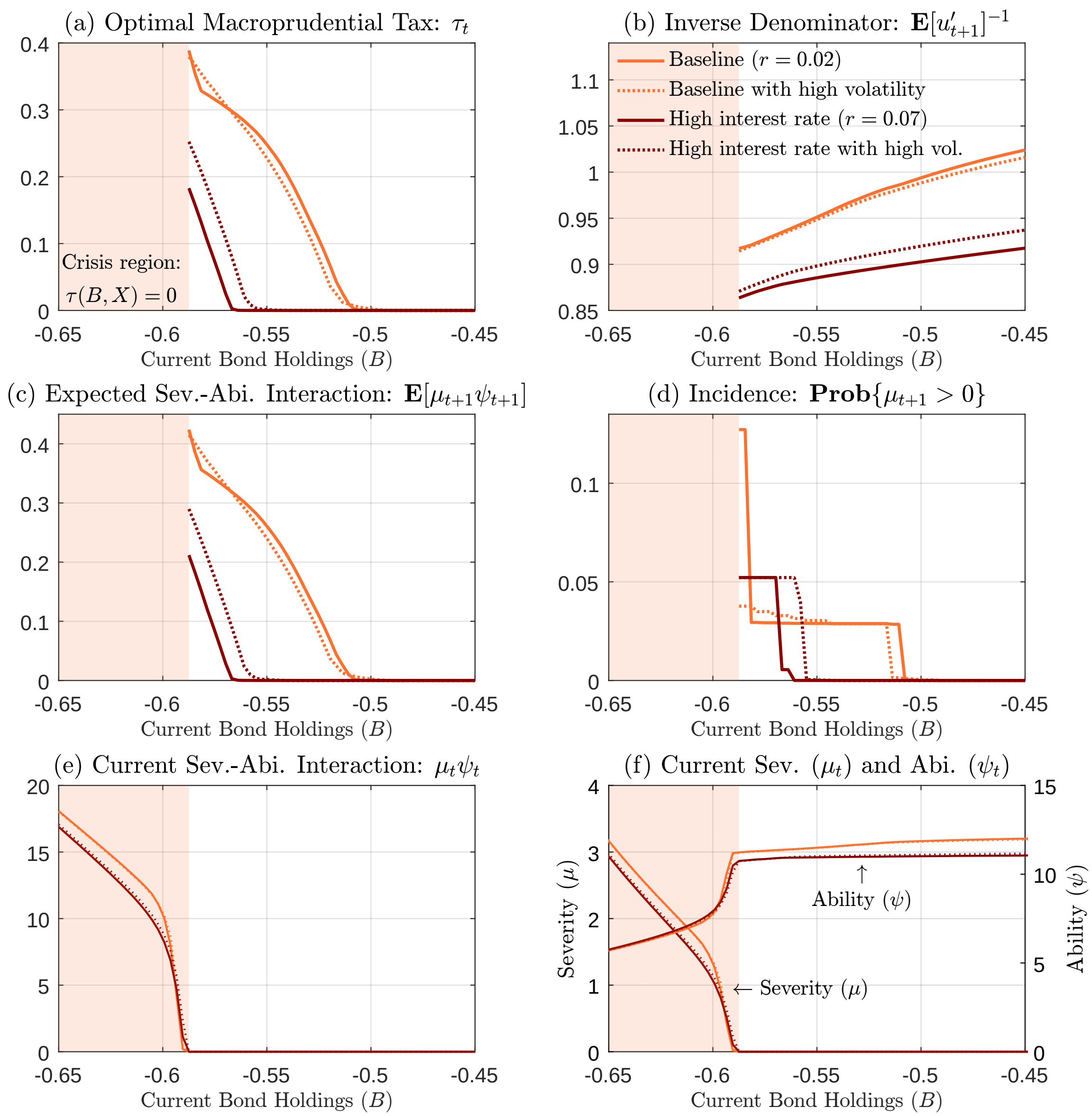
Loosely speaking, the optimal tax in (20) is determined by the expectation of future differences between the social and private marginal values of wealth, $\mathbb{E}[\kappa\psi(B', X')\mu(B', X')|X]$, which is expressed in units of consumption once divided by the expected marginal utility of consumption, $\mathbb{E}[u'(c(B', X'))|X]$. Note that we can also decompose the tax by focusing on the severity and the ability of the planner defined in Section 3. More precisely, let (B', X') denote any possible state of the economy tomorrow, and define the following terms: We refer to $\mu(B', X')$ as the *severity* of a future crisis, $\psi(B', X')$ as the *ability of the planner* to dampen the future crisis, and $\mathbf{Prob}\{\mu(B', X') > 0\}$ as the incidence of the crisis.

Figure 9 decomposes the optimal tax into the effects of the denominator in (20), $\mathbb{E}_t\left[u'(c_{t+1})\right]$, as well as those of the severity, ability of the planner, and incidence of future crises previously defined. Panel (a) plots the optimal tax as a function of current bond holding for the four different configurations of the exogenous states considered in Figure 7. Panel (b) plots the inverse of the expected private marginal value of an additional value of consumption tomorrow. These values remain relatively constant across bond levels as well as exogenous states, which implies that this part of the tax function does not fundamentally drive the difference in the effects of volatility shocks. Panel (c) plots the expected value of the interaction between the severity and the ability of the planner for future crises, which determines the numerator of the tax formula (20). Clearly, it is this expected interaction that drives the asymmetric effects of volatility shocks.

To further our understanding of the differences across tax functions, we can decompose the expected severity-ability interaction into the incidence of a future crisis (panel (d)) and the interaction between the severity and the ability of the planner in crises (panel (e)). From these two panels we immediately derive two results. First, given a configuration of the exogenous state of the economy, the shape of the optimal tax as a function of borrowing is primarily determined by the severity-ability interaction, the shae of which in turn is primarily driven by the shape of the severity of crises (see panel (f) of Figure 4). However, it is important to point out that the incidence also plays a large role in shaping this function, as can be seen from the shape of the tax function close to the crisis region for average interest rates and their volatility. Clearly, sudden changes in the shape of this function (convexity, concavity) are driven by drastic changes in the incidence of sudden stops, as in panel (d). Second, and focusing on the effects of volatility, these effect are primarily driven by differences in the incidence of future crises across different volatility regimes. Therefore, we now shift our attention to trying to understand the changes in the incidence of future crises depicted in panel (d).

The key to understanding the difference in the effects of the volatility shocks rests in understanding their non-monotonic effect on the incidence depending on the current levels of the interest rate. Note that, for average levels of interest rates, higher volatility leads to a decline in the incidence of future crises, except for values of current bond holdings in the interval $\mathcal{R} = [-0.58, -0.54]$. We can think of the incidence of future crises as determined by two forces: (i) endogenous changes in agents' decisions and (ii) exogenous changes in the probability of reaching more extreme states of the world. For values of bond holdings outside $\mathcal{R}$, higher volatility generates enough additional savings by households (endogenous changes in decisions) such that these savings counteract the increase in the exogenous probability of hitting the collateral constraint and being in worse states of the world. Hence, the SP imposes a lower tax on borrowing. For values of bond holdings contained in $\mathcal{R}$, the increase in savings due to greater volatility is not enough to account for the increase in the probability of being in a sudden stop.

Figure 9: Decomposition of Optimal Tax on Debt



Note: Panel (a) is equivalent to Figure 7, implying that exogenous state levels correspond to those previously specified. Panels (b) and (c) plot the inverse of the denominator and the numerator of equation (20), respectively. Panels (d), (e), and (f) provide additional details on the forces shaping the functions in panel (c).

Hence, the incidence of crises increases for high volatility, leading to a lower tax on debt.

Note that the same forces shaping the increase in the tax given a higher volatility in region $\mathcal{R}$ for average interest rates apply in the case of high interest rates. As previously discussed, for high enough interest rates, higher volatility can actually generate more borrowing by households. Therefore, endogenous decisions by agents reinforce the increase in the exogenous probability of being in worse states of the world, and the incidence of crises (dark brown lines in panel (f)) increases for higher volatility across all levels of borrowing. Hence, for high enough levels of interest rates, higher volatility of interest rates calls for higher macroprudential taxes on international borrowing.

To summarize, the fact that changes in the volatility of interest rates affect the endogenous behavior of agents as well as the exogenous probability of reaching certain states of the world in different and complex directions leads to non-monotonic changes and shifts in optimal tax functions. Hence, the task of counteracting the effects of volatility shocks on a small open economy is much more complex than a policy maker might think.

5 Conclusion

This paper explores the effects of shocks to the level and volatility of interest rates for economic activity in EMEs, particularly during sudden stop episodes. We extended the small open economy framework of Jeanne and Korinek (2018) to include these types of shocks in a spirit similar to Fernández-Villaverde et al. (2011). Three main lessons can be drawn from our analysis. First, changes in the level and volatility of market sentiment that is orthogonal to EMEs' economic fundamentals affect economic activity in these countries. Second, the dynamics of interest rate levels and volatility around sudden stops generated by the model have behavior similar to those observed empirically for EMEs. Third, the optimal macroprudential tax on foreign borrowing is a complex object that should respond to changes in the level and volatility of interest rates; however, quantitatively, increasing taxes is seldom optimal when volatility increases. This last result, which goes against many policymakers' conventional wisdom, is partially driven by agents' precautionary savings behavior induced by "macroprudential" volatility.

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