A Market-Based Term Structure of Expected Inflation

M. Ramos-Francia, S. García-Verdú and M. Sánchez-Martínez

Discussion by Ricardo Reyes-Heroles

Federal Reserve Board

ITAM Alumni Conference 2019

August 9, 2019

Overview (1)

- **What?** Rely on information on asset prices that depend on inflation to back out a term-structure of inflation expectations (market-based)

Overview (1)

- **What?** Rely on information on asset prices that depend on inflation to back out a term-structure of inflation expectations (market-based)

- **Why?** *Why do we care?* Inflation expectations are key to the decision making process of market participants, both private and *public*.

Overview (1)

- **What?** Rely on information on asset prices that depend on inflation to back out a term-structure of inflation expectations (market-based)

- **Why?** *Why do we care?* Inflation expectations are key to the decision making process of market participants, both private and *public*.

  Why this way? $\rightarrow$ Approaches to obtain expected inflation:

  1. Econometric model using inflation (direct)
  2. Survey-based measurements (direct)
  3. Market-based expectations (indirect)

  both 1. and 2. rely heavily on macro data

  $\Rightarrow$ *Macro data is a mess*: measurement error, frequency availability (e.g. monthly, quarterly,...), published with delays, subject to revisions,...

Overview (1)

- **What?** Rely on information on asset prices that depend on inflation to back out a term-structure of inflation expectations (market-based)

- **Why?** *Why do we care?* Inflation expectations are key to the decision making process of market participants, both private and *public*.

  Why this way? $\rightarrow$ Approaches to obtain expected inflation:

  1. Econometric model using inflation (direct)
  2. Survey-based measurements (direct)
  3. Market-based expectations (indirect)

  both 1. and 2. rely heavily on macro data

  $\Rightarrow$ *Macro data is a mess*: measurement error, frequency availability (e.g. monthly, quarterly,. . .), published with delays, subject to revisions,. . .

  $\rightarrow$ 3. circumvents these issues, but usually does not account for risk premia.

Overview (2)

- **How?** Recover expected inflation as difference of nominal and real (i.e. inflation-linked) interest rates under the assumption of risk neutral investors.

Overview (2)

- **How?** Recover expected inflation as difference of nominal and real (i.e. inflation-linked) interest rates under the assumption of risk neutral investors. In particular,

  1. Rely on *different* affine interest rate models to estimate nominal and real interest rates for different maturities separately (no use of data on inflation)
  2. Recover daily term structure of nominal and real interest rates associated with *risk neutral* investors,
  3. Analyze statistical relationship between market-based expectations and other measures.

Overview (2)

- **How?** Recover expected inflation as difference of nominal and real (i.e. inflation-linked) interest rates under the assumption of risk neutral investors. In particular,

   1. Rely on *different* affine interest rate models to estimate nominal and real interest rates for different maturities separately (no use of data on inflation)
   2. Recover daily term structure of nominal and real interest rates associated with *risk neutral* investors,
   3. Analyze statistical relationship between market-based expectations and other measures.

- **What has been done and how is this different?**
   - Abrahams, Adrian, Crump & Moench (2016): estimate affine term structure model to decompose real and nominal bond yields → use of data on inflation
   - Aguilar-Argaez, Elizondo,& Roldán-Peña (2016): study evolution of break-even inflation → focus on inflation premia

Asset Pricing 101: A Macro Approach

- Starting from first principles with a simpler problem:

$$\max_{c_t, b_t, b_t^{\$}, s_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ s.t.}$$

$$c_t + \frac{1}{1+i_t}\frac{b_t^{\$}}{p_t} + \frac{b_t}{1+r_t} + q_t s_t = \frac{b_{t-1}^{\$}}{p_t} + b_{t-1} + (d_t + q_t) s_{t-1}$$

Asset Pricing 101: A Macro Approach

- Starting from first principles with a simpler problem:

$$\max_{c_t, b_t, b_t^{\$}, s_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ s.t.}$$

$$c_t + \frac{1}{1+i_t}\frac{b_t^{\$}}{p_t} + \frac{b_t}{1+r_t} + q_t s_t = \frac{b_{t-1}^{\$}}{p_t} + b_{t-1} + (d_t + q_t) s_{t-1}$$

- Optimality conditions:

$$1 + i_t \approx (1 + r_t)\, \mathbb{E}_t [1 + \pi_{t,t+1}] \left(1 - cov_t \left(M_{t+1}, \frac{p_t}{p_{t+1}}\right)(1 + i_t)\right)$$

where $M_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the SDF.

Asset Pricing 101: A Macro Approach

- Starting from first principles with a simpler problem:

$$\max_{c_t, b_t, b_t^{\$}, s_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \text{ s.t.}$$

$$c_t + \frac{1}{1+i_t} \frac{b_t^{\$}}{p_t} + \frac{b_t}{1+r_t} + q_t s_t = \frac{b_{t-1}^{\$}}{p_t} + b_{t-1} + \left(d_t + q_t\right) s_{t-1}$$

- Optimality conditions:

$$1 + i_t \approx \left(1 + r_t\right) \mathbb{E}_t \left[1 + \pi_{t,t+1}\right] \left(1 - cov_t\left(M_{t+1}, \frac{p_t}{p_{t+1}}\right)\left(1 + i_t\right)\right)$$

where $M_{t+1} = \beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}$ is the SDF.

- If investors are risk-neutral, M_{t+1} is a constant and

$$1 + i_t \approx \left(1 + r_t\right) \mathbb{E}_t \left[1 + \pi_{t,t+1}\right]$$

Going from macro to finance...

- If one could observe the returns i_t and r_t on the portfolios of risk-neutral investors in isolation, then

$$\mathbb{E}_t\left[1+\pi_{t,t+1}\right]=\frac{1+i_t}{1+r_t},$$

but this would be a *heroic* assumption.

Going from macro to finance...

- If one could observe the returns i_t and r_t on the portfolios of risk-neutral investors in isolation, then

$$\mathbb{E}_t\left[1+\pi_{t,t+1}\right]=\frac{1+i_t}{1+r_t},$$

but this would be a *heroic* assumption.

- Instead, use asset-pricing equilibrium conditions

$$\mathbb{E}_t[M_{t+1}\left(1+r_t\right)]=1, \mathbb{E}_t[M^{\$}_{t+1}(1+i_t)]=1, \mathbb{E}_t[M_{t+1}\left(\frac{d_{t+1}+q_{t+1}}{q_t}\right)]=1$$

where $M^{\$}_{t+1}=M_{t+1}\frac{p_t}{p_{t+1}}$, to estimate SDFs and asset returns ($R_{j,t}$ for asset j) such that risk-neutral returns can be recovered as a special case of the general estimation

$$R_{j,t}=g_j(F_t;\lambda)\Rightarrow R^*_{j,t}=g_j(F_t;0)$$

where $*$ denotes risk-neutral $\rightarrow$ set loadings on risk equal to zero

Analysis: Nominal Bond Yields

- Standard decomposition for a nominal nth-period bond yield:

$$y^{\$}_{t,n} = \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1}}_{\text{ex-ante real short rates}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i}}_{\text{expected inflation}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t rx^{\$}_{r\to t+1,n-i+1}}_{\text{nominal term premium}}$$

- Paper is after: $\frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i}$ for different values of n

Analysis: Nominal Bond Yields

- Standard decomposition for a nominal nth-period bond yield:

$$y^{\$}_{t,n} = \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1}}_{\text{ex-ante real short rates}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i}}_{\text{expected inflation}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t rx^{\$}_{r\to t+1,n-i+1}}_{\text{nominal term premium}}$$

- Paper is after: $\frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i}$ for different values of n
- **Challenge:** Needs $\frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1}$ & $\frac{1}{n}\sum_{i=1}^{n} E_t rx^{\$}_{t\to t+1,n-i+1}$
  - $E_t rx^{\$}_{t\to t+1,n}$ is the one period expected excess return:

$$E_t rx^{\$}_{t\to t+1,n} + \frac{1}{2} V_t rx^{\$}_{t\to t+1,n} = -cov_t(m^{\$}_{t+1}, rx^{\$}_{t\to t+1,n})$$

and $m^{\$}_{t+1}$ is the nominal log stochastic discount factor.

Analysis: Real Bond Yields

- Standard decomposition for a real nth-period bond yield:

$$y_{t,n} = \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1}}_{\text{ex-ante real short rates}} + \underbrace{0}_{\text{expected inflation}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t rx_{r\rightarrow t+1,n-i+1}}_{\text{real term premium}}$$

  - $E_t rx_{t\rightarrow t+1,n}$ is the one period real expected excess return:

$$E_t rx_{t\rightarrow t+1,n} + \frac{1}{2} V_t rx_{t\rightarrow t+1,n} = -cov_t(m_{t+1}, rx_{t\rightarrow t+1,n})$$

  and m_{t+1} is the real log stochastic discount factor (SDF).

- Close link between the nominal $m_{t+1}^{\$}$ and real m_{t+1} SDF's

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$$

(more on this later)

This paper's novel insight and approach...

1. Use nominal yields $y^{\$}_{t,n}$ to estimate:

$$y^{\$}_{t,n} = \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1} + \frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i}}_{\text{i) ex-ante nominal short rates}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t rx^{\$}_{r\rightarrow t+1,n-i+1}}_{\text{ii) nominal term premium}}$$

2. Use real yields $y_{t,n}$ to estimate:

$$y_{t,n} = \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1}}_{\text{iii) ex-ante real short rates}} + \underbrace{\frac{1}{n}\sum_{i=1}^{n} E_t rx_{r\rightarrow t+1,n-i+1}}_{\text{iv) real term premium}}$$

3. Take the difference between i) and iii) to get $\frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i}$

No need to use macro data!!!

Comment # 1

1. Issues with the data:
   - Maturity for nominal yields $y^{\$}_{t,n}$:

     $$n = \{1 \text{ month}, \quad 1 \text{ year}, \quad 5 \text{ years}, \quad 10 \text{ years}, \quad 20 \text{ years}\}$$

   - Maturity for real yields $y_{t,n}$:

     $$n = \{1 \text{ month}, \quad 1 \text{ year}, \quad 5 \text{ years}, \quad 10 \text{ years}, \quad 20 \text{ years}\}$$

$\rightarrow$ No data on the short end of the real yield curve.

Comment # 1: How the authors solve this data limitation?

They make/need one of either two possible assumptions:

1. Nominal and real term premiums are negligible for small n

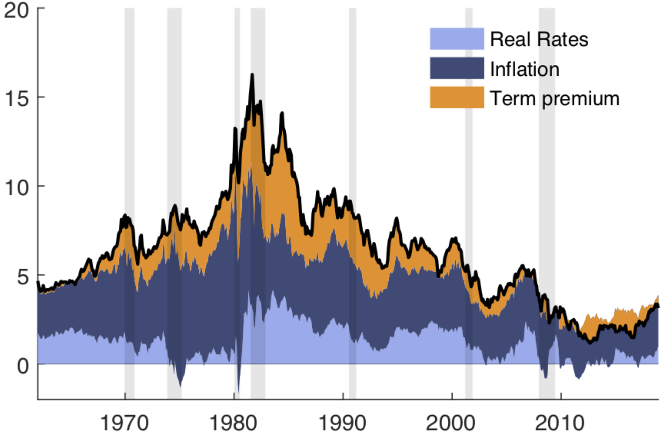
$$\underbrace{tp^{\$}_{t,n} = \frac{1}{n}\sum_{i=1}^{n} E_t rx^{\$}_{t\rightarrow t+1,n-i+1}}_{\text{nominal term premium}} \approx 0 \qquad \underbrace{tp_{t,n} = \frac{1}{n}\sum_{i=1}^{n} E_t rx_{t\rightarrow t+1,n-i+1}}_{\text{real term premium}} \approx 0$$

2. Nominal and real term premiums are equal for small n

$$tp^{\$}_{t,n} = tp_{t,n}$$
$$\downarrow$$
$$\underbrace{\text{inflation} + \text{liquidity} + \text{growth} + \text{others}}_{\text{nominal term premium}} = \underbrace{\text{liquidity} + \text{growth} + \text{others}}_{\text{real term premium}}$$
$$\downarrow$$
$$\text{inflation risk} = 0$$

Comment # 1: Are short term premiums small?

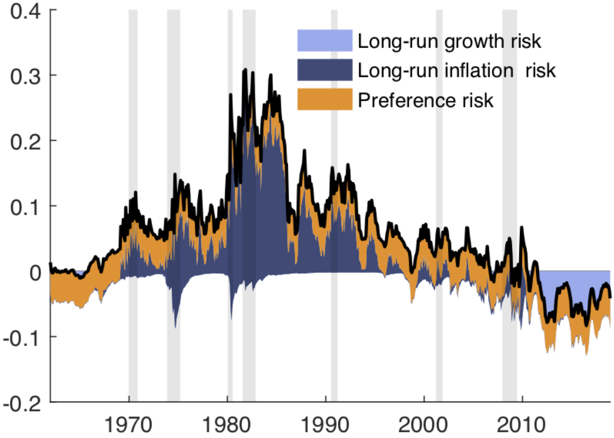
Decomposition of a 1 year nominal bond yield
Gómez-Cram and Yaron (2017)



The contribution of the term premium increases with maturity and for short yields contributes ≈ 13%

Comment # 1: Are short term inflation risks small?

Decomposition of a 1 year nominal expected excess bond returns
Gómez-Cram and Yaron (2017)



The contribution of inflation risks has decreased over time
but they can be quite high $\approx 80\%$ in the 80's

Comment # 1: How the authors solve this data limitation?

1. Nominal and real term premiums are negligible for small n?

   → Not so small!

2. Nominal and real term premiums are equal for small n?

   inflation risk ≈ 0?

   → Yes for the latter part of the sample
   → but they can be quite big

   $\Longrightarrow$ time varying bias in their measure of expected inflation:
   $\frac{1}{n}\sum_{i=1}^{n} E_t\pi_{t+i} + \text{inflation risk}_t$

Comment # 2

To estimate the nominal and real term premium the authors assume two SDF's

- Nominal SDF: $m^{\$}_{t+1} \rightarrow$ price nominal yields
- Real SDF: $m_{t+1} \rightarrow$ price real yields

$\rightarrow$ However, theory provides a link between $m^{\$}_{t+1}$ and m_{t+1}

$$m^{\$}_{t+1} = m_{t+1} - \pi_{t+1}$$

Model implied inflation rate:

$$\pi_{t+1} = m_{t+1} - m^{\$}_{t+1}$$

$\Longrightarrow$ It would be interesting to see this series!!!

Wrapping up. . .

- Very interesting paper! $\rightarrow$ potential to be well published
- What's next? Make it 'sexier'
  - How monetary policy (MEX or US) affects the term structure of inflation expectations? MEX response to US. . .
  - Big advantage of this methodology is that measure can be computed at very high frequencies. Event window type of approach or follow Romer and Romer (2004)