Sufficient Statistics for Dynamic Spatial Models

Kleinman, Liu & Redding (2021)

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NBER Summer Institute (ITI)

July 15, 2021

Overview 1/2

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- **Why?** Understand the response of spatial distribution of economic activity $\rightarrow$ incorporating forward-looking investment decisions is challenging
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  - How does the degree of capital-labor subs./compl. shape dynamics?

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- **Why?** Understand the response of spatial distribution of economic activity
  $\rightarrow$ incorporating forward-looking investment decisions is challenging
  - Need framework useful for quantitative analysis, but tractable
  - How does the degree of capital-labor subs./compl. shape dynamics?
- **How?** Introduce costly migration decisions (Caliendo et al., 2019) and 'real' trade (Armington) into multiregion neoclassical growth model
  - Assume hand-to-mouth workers and immobile capitalists
  - Consider log-linear approximations

$\rightarrow$ Log-linearize model to study effects of small shocks on steady states and transitional dynamics.

$\rightarrow$ *Empirical* application: take model to data of U.S. states from 1965-2015

- Disentangle convergence forces versus shocks along transition for distribution of economic activity

Overview 2/2

- **Novel qualitative results:**
  - Provide baseline tractable framework with forward-looking investment
  - Log-linear approximation delivers closed-form solutions for steady-state, impact, and transitional elasticities
- **Empirical application:**
  - Log-linear approximation is close to nonlinear solution
  - Convergence to steady state determined in the 60s predicts transition $\Rightarrow$ shocks since then not the most relevant?
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$\rightarrow$ Great contribution! Clean framework, clearly explained, and easy to implement. A novel tool definitely worth keeping handy in the toolbox!

Outline

1. Dynamic Block of the Model
2. Investment-Migration Interaction
3. Log-linearization
4. Two comments and food for thought

The Dynamic Block of the Model

Locations $i \in \{1, \ldots, N\}$ inhabited by mobile hand-to-mouth workers and immobile capitalists:

- **Workers:** (financial wealth of workers is not an endogenous state variable)

$$v_{it} = \ln\left(\frac{w_{it}}{p_{it}}\right) + \ln b_{it} + \rho \ln \sum_{g=1}^{N} \left(\frac{\exp\left(\beta v_{gt+1}\right)}{\kappa_{git}}\right)^{\frac{1}{\rho}}$$

$$\Rightarrow D_{igt} = \frac{\left(\exp\left(\beta v_{gt+1}\right) / \kappa_{git}\right)^{\frac{1}{\rho}}}{\sum_{m=1}^{N}\left(\exp\left(\beta v_{mt+1}\right) / \kappa_{mt}\right)^{\frac{1}{\rho}}}$$

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$$\max_{c_{it}, k_{it+1}} \sum_{t=0}^{\infty} \beta^t \ln c_{it} \quad \text{s.t.} \quad k_{it+1} = \underbrace{\left(\frac{r_{it}}{p_{it}} + (1-\delta)\right)}_{R_{it}} k_{it} - c_{it}$$

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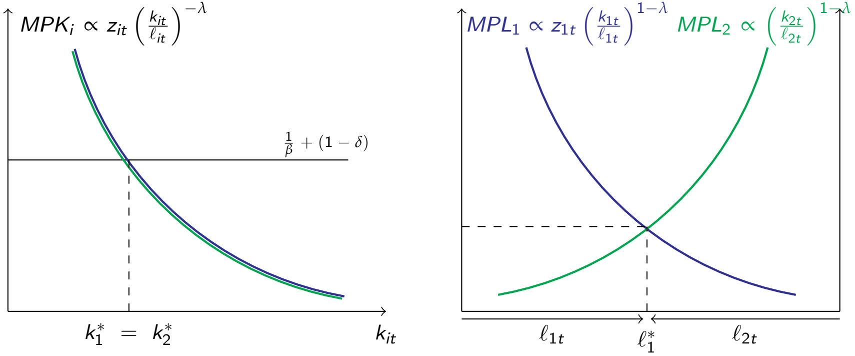
$\rightarrow$ Good to go! Solve sequence of static equilibria given states $\{k_{it}, \ell_{it}\}_{i=1}^{N}$.

Mechanism: Investment-Migration Interaction

Consider two symmetric locations in an initial steady state $\rightarrow$ "hand-wavy"



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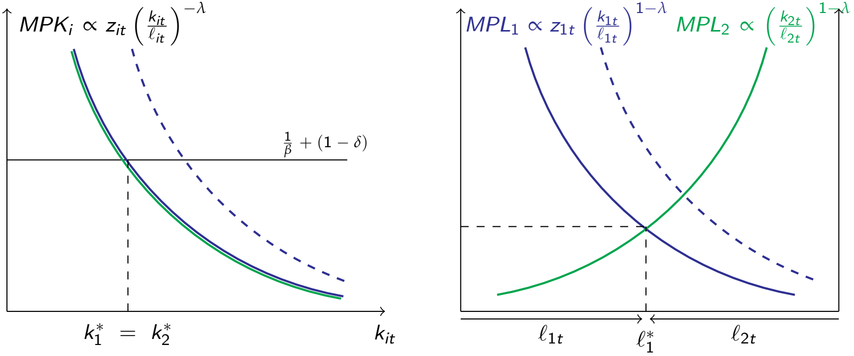

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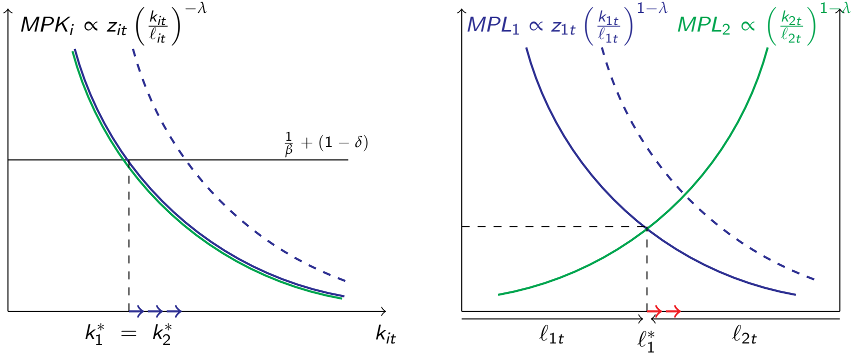

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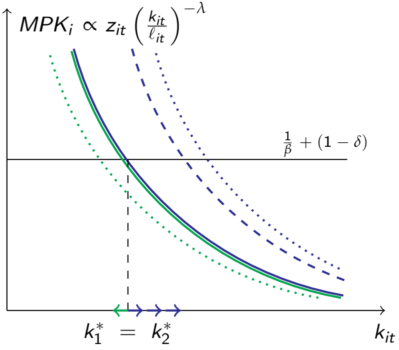

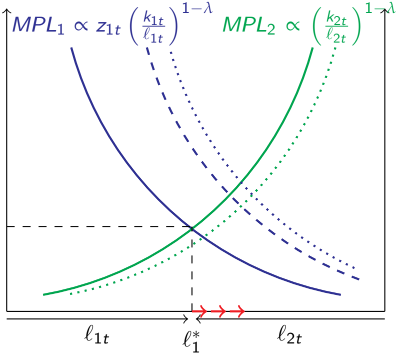

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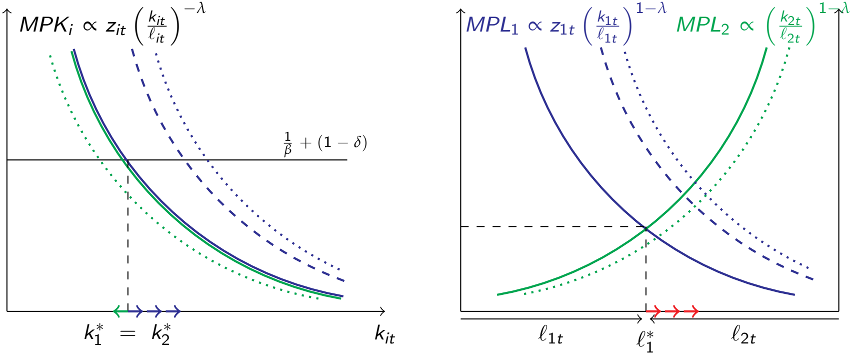

- Effects of shock on investment are amplified by migration because of capital-labor complementarity.
- Further away from SS ⇒ larger second-round and further effects

Log-linear Approximation

Even with simplifying assumptions

- No closed-form solutions for objects of interest (elasticities) and
- Solving nonlinear model by "shooting" (Ext. Path) is computationally intensive.

Relying on log-linear approximation:

P2 Steady-state elasticities depend on parameters and observed bilateral matrices ($\mathbf{S},\mathbf{T},\mathbf{D},\mathbf{E}$).

P3 One-time permanent shocks ($\tilde{\mathbf{f}} = \begin{bmatrix} \tilde{\mathbf{z}} \\ \tilde{\mathbf{b}} \end{bmatrix}$), then

$$\tilde{\mathbf{x}}_{t+1} = \mathbf{P}\tilde{\mathbf{x}}_t + \mathbf{R}\tilde{\mathbf{f}} \quad \text{for} \quad t \geq 1$$

where $\tilde{\mathbf{x}}_t \equiv \ln \mathbf{x}_t - \ln \mathbf{x}^*_{initial}$ and $\mathbf{P},\mathbf{R}$ depend on $\mathbf{S},\mathbf{T},\mathbf{D},\mathbf{F}$ at $t = 0$.

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Technical question: Log-linearize around *initial* steady state and use *observed* bilateral matrices ⇒ Does this imply that changes in bilateral matrices are of second-order for dynamics given small shocks?

Comment 1/2: Assumptions

Could some of the key assumptions underlying the proposed framework generate substantial differences in results?

- What if workers could save?
  - Profits of capitalists no longer linear in k?
  - Smooth consumption during adjustment process $\rightarrow$ first order effects on reallocation (Dix-Carneiro, Pessoa, Traiberman & Reyes-Heroles, 2020)
  - Wealth effects $\rightarrow$ Steady state no longer unique
- What if capitalists could move?
  - Valuation effects $\rightarrow$ What is the right price to value capital that can be moved across regions? (Ferriere, Navarro & Reyes-Heroles, 2020)
- What if capitalists could save?
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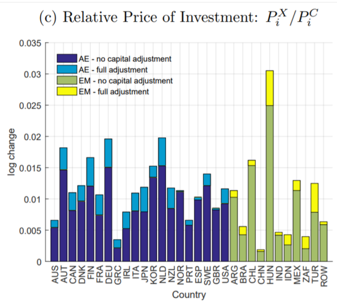
→ **Would we expect any of these issues to be key for transitional dynamics across space? Is it worth incorporating them in exchange for tractability?**

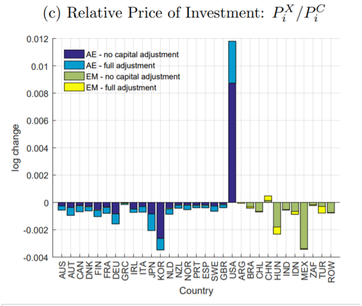
Comment 2/2: Ingredients → Price of Investment Goods

Multiple extensions considered → Difference between p^c_{it} and p^x_{it} seems of firs-order importance to me as soon as you have more than one sector

- **Why?** Sectoral composition of final C vs I is very different!

→ Reyes-Heroles, Traiberman & Van Leemput (2020): 5% global TW (left), U-C TW (right)





$$\Rightarrow \boxed{k_{it+1} = \left(r_{it}/p^x_{it} + (1-\delta)\right) k_{it} - p^c_{it}/p^x_{it} c_{it}}$$ where p^x_{it} is more intensive in tradables

Other Small Comments and More Food For Thought

- Shocks to specific bilateral trade and migration costs $\rightarrow$ log-linearization remains good approximation? (Kleinman, Liu & Redding, 2021)
- Irreversibility of capital
- Capital-skill complementarity
- Why not just focus on the Euler equation of the capitalists independently of preferences given that you are log-linearizing anyway?
- Focus on rational expectations equilibria and compare expected vs unexpected shocks
- Regional specialization and sectoral shocks